Notes on Synthetic Division:

How to factor polynomials (with integer coefficients)

Let $p(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1} x + a_n$

where a_i's are integers.

Try $\dfrac{\pm \text{ factors of } a_n}{\pm \text{ factors of } a_0}$ (usually $a_0 = 1$)

$$
\begin{array}{c|ccccc|c}
r & a_0 & a_1 & a_2 & \cdots\cdots & & a_n \\
 & & + & + & & & \\
 & \downarrow & r a_0 & r a_1 + r^2 a_0 & & & \\
\hline
 & a_0 & a_1 + r a_0 & a_2 + r a_1 + r^2 a_0 & & &
\end{array}
$$

remainder.

If r is a root, then remainder is zero

1 $p(x) = x^3 - 2x^2 - 5x + 6$

Try

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -5 & +6 \\ & & -1 & 3 & 2 \\ \hline & 1 & -3 & -2 & 8 \end{array}$$

remainder $\neq 0$ (not good).

in fact:

$$x^3-2x^2-5x+6 = (x+1)(x^2-3x-2)+8$$

Try again

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & 6 \\ \hline -2 & 1 & -1 & -6 & 0 \\ & & -2 & 6 & \\ \hline 3 & 1 & -3 & 0 & \\ & & 3 & & \\ \hline & 1 & 0 & & \end{array}$$

$$\begin{aligned} & x^3-2x^2-5x+6 \\ & \quad \| \\ & (x-1)(x^2-x-6) \\ & \quad \| \\ & (x-1)(x+2)(x-3) \end{aligned}$$

②

$$p(x) = x^3 - 3x^2 + 4$$

Try ↘

$$\begin{array}{r|rrr|r} 1 & 1 & -3 & 0 & 4 \\ & & 1 & -2 & -2 \\ \hline & 1 & -2 & -2 & 2 \end{array}$$

remainder $\neq 0$
(not good)

(in fact

$$x^3 - 3x^2 + 4 = (x-1)(x^2 - 2x - 2) + 2$$

Try again:

$$\begin{array}{r|rrr|r} 2 & 1 & -3 & 0 & 4 \\ & & 2 & -2 & -4 \\ \hline 2 & 1 & -1 & -2 & 0 \\ & & 2 & 2 & \\ \hline -1 & 1 & 1 & 0 & \\ & & -1 & & \\ \hline & 1 & 0 & & \end{array}$$

$$x^3 - 3x^2 + 4 = (x-2)(x^2 - x - 2) = (x-2)(x-2)(x+1)$$