Homework 2

MA 538 Spring 2009 (Aaron N. K. Yip)

Due: 3pm, Monday, Feb. 23

From Textbook (Billingsley):

page 35: # 2.19

page 64: # 4.4, 4.5, 4.11(d,e), 4.12

Additional Problems

1. Given a sequence of events $C_1, C_2, \ldots$. Suppose $\lim_n P(C_n) = 0$ and $\sum_n P(C_{n+1} \backslash C_n) < \infty$. Prove that $P(\limsup_n C_n) = 0$.

2. Consider the coin tossing experiment with a fair coin. Let L_n be the length of the n-th block of (a continuous segment of) 0's. For any $\epsilon > 0$, find, if possible, outer and inner boundaries $\{r_n\}$ and $\{s_n\}$ such that $\limsup_n \frac{r_s}{s_n} \leq 1 + \epsilon$. For simplicity, you can assume that the r_n, s_n's converge to ∞ monotonically as $n \longrightarrow \infty$.

   (Recall that r_n's is an outer boundary for the L_n's if $P(L_n > r_n \ \text{i.o.}) = 0$ and s_n's is an inner boundary for the L_n's if $P(L_n > s_n \ \text{i.o.}) = 1$.)

3. Do the same as in the previous problem but for X_n's which are i.i.d random variables with the Cauchy distribution, i.e.

$$P(X \leq a) = \int_{-\infty}^{a} \frac{dx}{\pi(1 + x^2)}$$