

Homework 3

MA 538 Spring 2009 (Aaron N. K. Yip)

Due: 3pm, Monday, Mar. 9

From Textbook (Billingsley):

page 186: #13.1, 13.2, 13.6

page 220: #16.8

page 229: #17.4, 17.6, 17.7

page 239: #18.13, 18.14

Additional Problems

1. In class, it is only proved that

$$\int \alpha f + \beta g dP = \alpha \int f dP + \beta \int g dP$$

for non-negative functions f and g and non-negative constants α and β .

Extend the above result for any integrable functions and constants.

2. Let $\{A_n\}_{n \geq 1}$ be a sequence of independent events. Recall the following set: $\liminf_n A_n = \{\omega : \text{there exists an } N(\omega) < \infty \text{ such that for all } n \geq N, \text{ then } \omega \in A_n\}$. For preciseness and without loss of generality, let N be the minimum number with the stated property.
 - (a) Give a necessary and sufficient condition for $P(\liminf_n A_n) = 1$ in terms of the $P(A_n)$'s.
 - (b) Find the distribution of N , i.e. find $P(N \leq k)$ for all k .
 - (c) Find a necessary and sufficient condition for $EN < \infty$ in terms of the $P(A_n)$'s.