Homework 4

MA 538 Spring 2009 (Aaron N. K. Yip)

Due: 3pm, Wed, Apr. 1

From Textbook (Billingsley):

page 270 (section 20): #14, 15, 16, 19, 21, 22

page 280 (section 21): #14, 20

Additional Problems

1. Give examples of sequence of random variables $\{X_n\}_{n=1}^{\infty}$ such that

   (a) X_n's converges almost surely but not in L^2-mean;

   (b) X_n's converges in L^2-mean but not almost surely;

   (c) X_n's converges in probability but not in L^2-mean.

2. Let $\{X_n\}_{n=1}^{\infty}$ be independent random variables with finite mean $EX_n = m_n$ and variance $\mathrm{Var}(X_n) = \sigma_n^2$. Suppose further that $\frac{1}{n}\sigma_n^2 \longrightarrow 0$ as $n \longrightarrow 0$, then

   $$\frac{1}{n}\left(S_n - \sum_{i=1}^{n} m_i\right) \longrightarrow 0 \quad \text{in } L^2\text{-mean}$$

   (As an application, this exercise strengthens the convergence in probability in the usual WLLN to convergence in L^2-mean, under even weaker assumptions.)