Problem correspondence:

Green:	1	2	3	4	5	6	7	8	9	10	11	12
Ans	E	B	E	B	A	C	D	D	B	A	C	C
Orange.	4.	1	2	5	3	6	12	7	8	9	10	11
Ans:	D	C	E	B	A	A	E	B	B	D	C	D

Sol (Based on green)

Orange: Orange exam numbers

Green: Green exam numbers.

1.
4.

$$\begin{cases} y' - 3\frac{y}{x} = x^2 \\ y(1) = 1 \end{cases} \qquad y(e) = ?$$

Linear.

$\Rightarrow \mu = e^{-\int \frac{3}{x} dx} = x^{-3}$

$\Rightarrow (y\mu)' = x^2 \cdot x^{-3} = \frac{1}{x}$

$\Rightarrow y\mu = \ln x + C$

$\Rightarrow y = x^3 \ln x + Cx^3$

$y(1) = 1 \Rightarrow C = 1$

$\Rightarrow y = x^3 \ln x + x^3$

$\Rightarrow y(e) = e^3 \ln e + e^3$

$= 2e^3$

2.
1.

$$\begin{cases} (1)\ \frac{dy}{dx} + 2y^4 = x^4 \qquad (2)\ y^3 \frac{dy}{dx} = \sin x \qquad (3)\ \frac{dy}{dx} = \sin x \\ y(0) = 0. \end{cases}$$

(1) $\Rightarrow \frac{dy}{dx} = x^4 - 2y^4$ (2) $\Rightarrow \frac{dy}{dx} = y^{-3}\sin x$ (3) $\Rightarrow \frac{dy}{dx} = \sin x$.

Theorem:

$\frac{dy}{dx} = f(x,y)$ uniquely solv

need $\begin{cases} \frac{\partial f}{\partial y} \\ f \end{cases}$ cont near initial data. $\Rightarrow$ (1) (3) ✓

(2) ×

3.

2

$$\begin{cases} \frac{dy}{dx} = 4xy^2 \\ y(0) = 1 \end{cases}$$

Seperable $\Rightarrow$ $\frac{dy}{y^2} = 4x\,dx$

$\Rightarrow$ $-\frac{1}{y} = 2x^2 + C$

$y(0) = 1 \Rightarrow C = -1$

$\Rightarrow$ $y = -\frac{1}{2x^2 - 1}$

4.

5

$$\frac{dy}{dx} = (x+y)^2 - 1$$

General solution ?

Not seperable , not linear. , not homogeneous. , not exact form.

Need to think substitution.

Let $v = x + y$ $\Rightarrow$ $\frac{dv}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow$ $\frac{dv}{dx} = v^2$

$\Rightarrow$ $\frac{dv}{v^2} = dx$

$\Rightarrow$ $-\frac{1}{v} = x + c$

$\Rightarrow$ $-\frac{1}{x+c} = x + y$

$\Rightarrow$ $y = -x - \frac{1}{x+c}$

5.

$$\frac{dy}{dx} = -\frac{2xy+1}{x^2+y}$$

3.

Find general solution

Not homogeneous, try exact form

$$\Rightarrow \quad \underbrace{(x^2+y)}_{N}\,dy + \underbrace{(2xy+1)}_{M}\,dx = 0$$

$$\frac{\partial N}{\partial x} = 2x \quad = \quad \frac{\partial M}{\partial y} = 2x$$

exact!

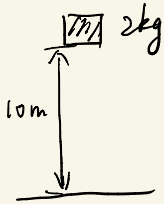
$$\Rightarrow \quad F = \int N\,dy + g(x) = x^2 y + \frac{1}{2}y^2 + g(x)$$

$$\frac{\partial F}{\partial x} = M \Rightarrow g(x) = x$$

$$\Rightarrow \quad x^2 y + \frac{1}{2}y^2 + x = C$$

6.

6



$V_0 = 0\,m/s$

$$a = g$$

$$\Rightarrow \frac{dv}{dt} = g$$

$$\Rightarrow v = gt + V_0 = \frac{dx}{dt}$$

$$\Rightarrow x = \frac{1}{2}gt^2 + V_0 t$$

$$= \frac{1}{2}gt^2$$

$$\Rightarrow t = \sqrt{\frac{20}{g}}$$

$$\Rightarrow V = gt = \sqrt{20g}$$

7.

$$A = \begin{pmatrix} 1 & 0 & c \\ 1 & 2 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & 0 \\ 2 & 4 \\ 1 & 0 \end{pmatrix}$$

12.

$$A^T = \begin{pmatrix} 1 & 1 \\ 0 & 2 \\ c & 0 \end{pmatrix} \qquad B^T = \begin{pmatrix} 2 & 2 & 1 \\ 0 & 4 & 0 \end{pmatrix}$$

$$\Rightarrow \quad A + B^T = \begin{pmatrix} 3 & 2 & c+1 \\ 1 & 6 & 0 \end{pmatrix} \qquad A^T + B = \begin{pmatrix} 3 & 1 \\ 2 & 6 \\ c+1 & 0 \end{pmatrix}$$

(A) X dimension not match

(B) X dimension not match

(C) X dimension not match $\qquad (AB)^T = B^T A^T$

$$AB(1,1) = 1 \times 2 + 0 \times 2 + c \times 1 = 2 + c = 1$$

$$\Rightarrow c = -1$$

(D) ✓

(E) X

8. $Ax = b$. Not true.

7

A. ✓

B. ✓

C. ✓

D. X $\qquad \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

E. ✓

9.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 5 & 3 \\ 0 & 1 & 2 \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad b = \begin{pmatrix} -3 \\ 9 \\ -2 \end{pmatrix} \quad x_3 = ?$$

8.

$$(A \mid b) \Longrightarrow \begin{pmatrix} 1 & 2 & 3 & -3 \\ 1 & 5 & 3 & 9 \\ 0 & 1 & 2 & -2 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 2 & 3 & -3 \\ 0 & 3 & 0 & 12 \\ 0 & 1 & 2 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 1 & 2 & -2 \end{pmatrix}$$

$$\to \begin{pmatrix} 1 & 2 & 3 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 2 & -6 \end{pmatrix} \to \begin{pmatrix} 1 & 2 & 3 & -3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -3 \end{pmatrix}$$

$$\Rightarrow x_3 = -3$$

10. $c = ?$ Infinity many soln?

9.

$$\begin{pmatrix} 3 & -2 & 5 & 1 \\ & 2 & 1 & 1 \\ -3 & 6 & c & 1 \end{pmatrix} \to \begin{pmatrix} 3 & -2 & 5 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 4 & 5+c & 2 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} 3 & -2 & 5 & 1 \\ 0 & 2 & 1 & 1 \\ 0 & 0 & 3+c & 0 \end{pmatrix}$$

$$\Rightarrow c = -3$$

11. $u = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $v = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $w = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

10. which is wrong statement?

A. $u \cdot v = 1 \times (-1) + 0 \times 1 = -1$ ✓

B. $u^T v = (1, 0)\begin{pmatrix} -1 \\ 1 \end{pmatrix} = -1$ ✓

$\qquad\qquad\qquad\qquad\qquad\qquad \parallel$

$v^T u = (-1, 1)\begin{pmatrix} 1 \\ 0 \end{pmatrix} = -1$

C. $\{u, v, w\}$ linearly independent X

D. $\{v, w\}$ linearly dependent ✓ $w = -2v$.

E. w is a linear combination of u and v. ✓.

12. $T(x) = Ax$ $A = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$ $u = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $v = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

11.

(i) $T(u) = Au = \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \neq \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ X

(ii) $T(2u + v) = 2T(u) + T(v)$ ✓.

(iii) $T(x) = Ax$ ✓.