MA 26200, Spring 2020, Exam 1
Version 02 (Green)

INSTRUCTIONS

(1) **Switch off your phone upon entering the exam room.**

(2) **Do not open the exam booklet until you are instructed to do so.**

(3) **Before you open the booklet, fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**

(4) **MARK YOUR TEST NUMBER ON THE SCANTRON**

(5) Once you are allowed to open the exam, make sure you have a complete test. There are 6 different test pages, including this cover page.

(6) Do any necessary work for each problem on the space provided or on the back of the pages of this booklet. Circle your answers in the booklet.

(7) Use a # 2 pencil to transcribe your answers to the scantron.

(8) After you have finished the exam, hand in your scantron and your test booklet to your recitation instructor.

RULES REGARDING ACADEMIC DISHONESTY

(1) Do not leave the exam during the first 20 minutes of the exam.

(2) Do not leave in the last 10 minutes of the exam.

(3) No talking. Do not seek or obtain any kind of help from anyone to answer the problems on the exam. If you need assistance, consult an instructor.

(4) Do not look at the exam of another student. You may not compare answers with other students until your exam is finished and turned in, and then only after you have left the room.

(5) Your bags must be closed throughout the exam period.

(6) Notes, books, calculators and phones must be in your bags and cannot be used.

(7) Do not handle phones or cameras or any other electronic device until you have finished and turned in your exam, and then only if you have left the room.

(8) When time is called, all students must put down their writing instruments immediately. You must remain in your seat while the TAs will collect the exam booklets and the scantrons.

(9) Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for such behavior can be severe and may include an automatic F on the course. All cases of academic dishonesty will be reported to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME   _____

STUDENT SIGNATURE   _____

STUDENT PUID   _____

SECTION NUMBER   _____

RECITATION INSTRUCTOR   _____

1. If $y(x)$ with $x > 0$ satisfies $y' - 3\frac{y}{x} = x^2$ and $y(1) = 1$, then $y(e) =$

    A. e^4

    B. e^{-2}

    C. $2e^{-3}$

    D. e^3

    E. $2e^3$

2. Consider the following three differential equations:

$$\text{(i) } \frac{dy}{dx} + 2y^4 = x^4, \quad \text{(ii) } y^3\frac{dy}{dx} = \sin x, \quad \text{(iii) } \frac{dy}{dx} = \sin x,$$

with initial condition $y(0) = 0$. Which of these problems has a unique solution in some interval $a < x < b$ containing 0 because we can apply the Existence and Uniqueness of Solutions Theorem?

    A. (i) and (ii) only

    B. (i) and (iii) only

    C. (ii) and (iii) only

    D. (i), (ii), and (iii)

    E. None of the three problems

3. The solution of $\frac{dy}{dx} = 4xy^2$, $y(0) = 1$ is:

A. $y = \left(\frac{1}{3x^2-1}\right)^{\frac{1}{3}}$

B. $y = \left(\frac{-1}{3x^2-1}\right)^{\frac{1}{3}}$

C. $y = e^{\frac{x^2}{2}}$

D. $y = \frac{1}{2x^2+1}$

E. $y = \frac{-1}{2x^2-1}$

4. Consider the equation
$$\frac{dy}{dx} = (x+y)^2 - 1,$$
which of the following is the general solution?

A. $-\frac{1}{x+c}$

B. $-\frac{1}{x+c} - x$

C. $\frac{1}{x+c} - x$

D. $-\frac{1}{x} + c$

E. $-\frac{1}{x} - x + c$

5. Find the general solution of the differential equation
$$\frac{dy}{dx} = -\frac{2xy + 1}{x^2 + y}.$$

A. $x^2 y + x + \dfrac{1}{2}y^2 = C$

B. $x^2 y + x = C$

C. $xy^2 + y + \dfrac{1}{3}x^3 = C$

D. $xy + \dfrac{1}{3}x^3 = C$

E. $xy^2 + y + \dfrac{1}{2}x^2 = C$

6. A ball of mass 2 kg is dropped from a height of 10 m above the ground in a vacuum. Assume the only force acting on the ball is gravity and the initial velocity is 0 m/sec. Let the constant of gravitation be g m/sec^2. Find the speed of the ball right before it hits the ground.

A. $\sqrt{20}$ m/sec

B. $\sqrt{40g}$ m/sec

C. $\sqrt{20g}$ m/sec

D. 400 m/sec

E. $20g$ m/sec

7. Let

$$A = \begin{bmatrix} 1 & 0 & c \\ 1 & 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 2 & 4 \\ 1 & 0 \end{bmatrix}.$$

Which of the following statements is true?

A. $A + B^T = A^T + B$

B. $AB = BA$

C. $(AB)^T = A^T B^T$

D. If the $(1,1)$-entry of AB is 1, then $c = -1$

E. If the $(1,1)$-entry of AB is 1, then $c = 1$

8. If $A\mathbf{x} = \mathbf{b}$, where A is a matrix and $\mathbf{x}$ and $\mathbf{b}$ are vectors, which of the following statements is NOT necessarily true?

A. If A has 3 columns, then $\mathbf{x}$ has 3 entries

B. If A has 5 rows, then $\mathbf{b}$ has 5 entries

C. If $\mathbf{x} = \mathbf{0}$, then $\mathbf{b} = \mathbf{0}$ regardless of the entries of A

D. If $\mathbf{b} = \mathbf{0}$, then $\mathbf{x} = \mathbf{0}$ regardless of the entries of A

E. $\mathbf{b}$ is in the span of the columns of A

9. Suppose

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 5 & 3 \\ 0 & 1 & 2 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -3 \\ 9 \\ -2 \end{bmatrix}.$$

If $A\mathbf{x} = \mathbf{b}$, then $x_3 =$

A. -5

B. -3

C. 0

D. 2

E. 4

10. For which value of c does the following system have infinitely many solutions?

$$\begin{cases} 3x - 2y + 5z = 1 \\ 2y + z = 1 \\ -3x + 6y + cz = 1 \end{cases}$$

A. -3

B. -1

C. $1/2$

D. $5/2$

E. 2

11. Consider the following vectors

$$\mathbf{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}.$$

Which of the following statements is **wrong**?

A. $\mathbf{u} \cdot \mathbf{v} = -1$

B. $\mathbf{u}^T\mathbf{v} = \mathbf{v}^T\mathbf{u}$

C. $\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$ is linearly independent

D. $\{\mathbf{v}, \mathbf{w}\}$ is linearly dependent

E. $\mathbf{w}$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$

12. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation and $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ is the standard matrix for T. Let

$\mathbf{u} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Consider the following statements:

(i) $T(\mathbf{u}) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, (ii) $T(2\mathbf{u} + \mathbf{v}) = 2T(\mathbf{u}) + T(\mathbf{v})$, (iii) $T(\mathbf{x}) = A\mathbf{x}$ for any $\mathbf{x} \in \mathbb{R}^2$.

Which of the above statements is/are true?

A. (i) only

B. (i) and (ii) only

C. (ii) and (iii) only

D. (iii) only

E. (i), (ii), and (iii)