

Linear Equ:

#1.

$$\begin{cases} y' = x - y \\ y(0) = 1 \end{cases}$$

$$\Rightarrow y' + y = x$$

$$\mu(x) = e^{\int 1 dx} = e^x$$

$$\Rightarrow (\mu(x)y)' = \mu(x) \cdot x$$

$$\Rightarrow y = \frac{\int e^x x}{e^x} = \frac{x e^x - e^x + c}{e^x} = (x-1) + c e^{-x}$$

plug in $y(0) = 1$

$$\Rightarrow c = 2$$

$$\Rightarrow y = x - 1 + 2e^{-x}$$

$$\Rightarrow y(1) = 2e^{-1}$$

#2.

$$\begin{cases} \frac{dy}{dx} = \frac{2x(y-2)}{x^2+1} \\ y(0) = 4 \end{cases}$$

$$\Rightarrow \int \frac{dy}{y-2} = \int \frac{2x dx}{x^2+1}$$

$$\Rightarrow \ln|y-2| = \ln|x^2+1| + c$$

plug in $y(0) = 4 \Rightarrow c = \ln 2$

$$\Rightarrow |y-2| = 2(x^2+1)$$

$$\Rightarrow y(1) = 6$$