Quiz 4 solution

February 14, 2020

1 B

we have $(3, \alpha, \beta) \in span(\vec{v_1}, \vec{v_2})$, so there exists some number x_1 and x_2 make

$$x_1\vec{v_1} + x_2\vec{v_2} = (3, \alpha, \beta)$$

. Which gives

$$\begin{aligned} 5x_1 + x_2 &= 3 \\ 2x_1 &= \alpha \\ x_1 &= \beta \end{aligned}$$

Immediately, we have $\beta = 2\alpha$

2 c

Basically this question is asking about the linear dependence of the given three vectors. Which is asking do we have non-travail solution to the system

$$x_1u + x_2v + x_3w = 0$$

Which is asking about the solution of

$$\begin{bmatrix} u & v & w \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

refering back to the values of u,v, w in the problem, we are indeed solving this problem

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 1 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If we do the row-reduction to the matrix, we will found it goes to this form

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Which has all the pivot places in each column, and each row, given the right hand side vector is (0, 0, 0), we know this system have one unique solution, which is $x_i = 0$, for i= 1,2,3. So this system has only travail solution.

So u, v and w are linear independent. In this way, any vector could not be written as a linear combination of the other two (if so, say w = c*u + d*v for example, then we have c*u + d*v - w = 0, which says we have non-travail solution to $x_1u + x_2v + x_3w = 0$, and the sol is (c, d, -1), this contradicts what we have derived above.)