Quiz 6 solution

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1

Since we only want to compute (3,2) entry of A^{-1}. So if we compute the 2nd column of A^{-1}, we are done. Since $AA^{-1} = I$, so if we denote 2nd column of A^{-1} to be x, then

$$Ax = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

And now we only care 3rd entry of x, which is corresponding to the (3,2) entry of A^{-1}, and we already know $\det A = -10$, so we denote $b = (0\ 1\ 0)'$ and use Cramer's Rule $x_3 = \frac{\det A_3 b}{\det A}$

where, by definition, $A_3 b$ is the matrix by substituting 3rd column of A by b

$$A_3 b = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 5 & 1 \\ 3 & 4 & 0 \end{bmatrix}$$

Easy to compute $detA_3 b = (-1)^{(2+3)}(2 * 4 - 3 * 1) = -5$. Hence $x_3 = \frac{1}{2}$

2

$det(-2AB^{-1}) = det(-2IAB^{-1}) = det(-2I)det(A)det(B^{-1})$
Since $det(B)det(B^{-1}) = det(I) = 1$, we know $det(B^{-1}) = \frac{1}{det(B)}$. Importantly, $det(-2I) = (-2)^3 = -8$, so we only need to calculate $det(A)$ and $det(B)$.

And we can calculate $det(A) = -14$ and $det(B) = 4$. So $det(-2AB^{-1}) = det(-2I)det(A)/det(B) = -8 * (-14)/4 = 28$

Comment:
For $n \times n$ dimensional matrix A,
$det(-2A) \neq -2det(A)$
$det(-2A) = (-2)^n det(A)$ is the right way to compute