§12.6, P567, #10

$u(0,t) = 100^\circ C$, $u(L,t) = 0^\circ$, $u(x,0) = 100^\circ C$

$$u_t = c^2 u_{xx}$$

Let $u = \left(0 - 100\right)\frac{x}{L} + 100 + W(x,t) = 100 - \frac{100x}{L} + W(x,t)$

$\Rightarrow u(0,t) = 100^\circ C = 100 + W(0,t) \Rightarrow W(0,t) = 0^\circ C$

$u(L,t) = 0 = W(L,t) \Rightarrow W(L,t) = 0^\circ C$

$100 = u(x,0) = 100 - \frac{100}{L}x + W(x,0) \Rightarrow W(x,0) = \frac{100}{L}x$

$W_t = c^2 W_{xx}$

$$W(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L} e^{-\left(\frac{cn\pi}{L}\right)^2 t}$$

$$B_n = \frac{2}{L}\int_0^L \frac{100}{L}x \sin\frac{n\pi x}{L}\,dx$$

$$u(x,t) = \frac{100}{L}\left(L - x\right) + W(x,t)$$