#10 $\vec{y}' = \begin{bmatrix} -3 & -4 \\ 5 & 6 \end{bmatrix} \vec{y} + \begin{bmatrix} 5 \\ -6 \end{bmatrix} e^t$

homog. $0 = \begin{vmatrix} -3-\lambda & -4 \\ 5 & 6-\lambda \end{vmatrix} = (\lambda+3)(\lambda-6) + 20 = \lambda^2 - 3\lambda + 2$

$= (\lambda - 1)(\lambda - 2)$

$\lambda = 1$ $\quad 5x_1 + 5x_2 = 0, \quad x_1 + x_2 = 0 \quad \vec{x}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\lambda = 2$ $\quad 5x_1 + 4x_2 = 0, \quad \vec{x}_2 = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$

$\vec{y}_h = c_1 e^t \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 4 \\ -5 \end{bmatrix} + \vec{y}_p = e^t \begin{bmatrix} 1 \\ 0 \end{bmatrix} + te^t \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$\vec{y}_p = \underline{e^t} \vec{v} + \underline{te^t} \vec{u}$

$(te^t)' = e^t + te^t$

$= (1+t)e^t$

$$\vec{y}_p' = \cancel{e^t}\vec{v} + (1+t)\cancel{e^t}\vec{u} = A\left[\cancel{e^t}\vec{v} + t\cancel{e^t}\vec{u}\right] + \begin{bmatrix}5\\-6\end{bmatrix}\cancel{e^t}$$

$$\vec{v} + (1+t)\vec{u} = A\vec{v} + tA\vec{u} + \begin{bmatrix}5\\-6\end{bmatrix}$$

$$(A-I)\vec{u} = \vec{0} \qquad \begin{bmatrix}-4 & -4\\5 & 5\end{bmatrix} \qquad 5u_1 + 5u_2 = 0 \qquad u_2 = -u_1$$

$$\vec{u} = \begin{bmatrix}k\\-k\end{bmatrix}$$

$$(A-I)\vec{v} = \vec{u} - \begin{bmatrix}5\\-6\end{bmatrix} = \begin{bmatrix}k-5\\-k+6\end{bmatrix}$$

$$-4v_1 - 4v_2 = -4$$

$$v_1 + v_2 = 1$$

$$\begin{bmatrix}-4 & -4 & k-5\\5 & 5 & -k+6\end{bmatrix} \frac{5}{4} \rightarrow \begin{bmatrix}-4 & -4 & k-5\\0 & 0\end{bmatrix} \qquad \vec{v} = \begin{bmatrix}1\\0\end{bmatrix}$$

$$\frac{5k-25}{4} + \frac{-4k+24}{4} = \frac{k-1}{4} = 0 \Rightarrow k = 1 \qquad \vec{u} = \begin{bmatrix}1\\-1\end{bmatrix}$$