

§7.9 #4, P318 Solution

$$V = \left\{ A_{3 \times 3} \mid A^t = -A \right\}$$

$\forall A, B \in V$ and $\forall \alpha, \beta \in \mathbb{R}$

$$\begin{aligned} (\alpha A + \beta B)^t &= \alpha A^t + \beta B^t \\ &= -\alpha A - \beta B \\ &= -(\alpha A + \beta B) \end{aligned}$$

$\Rightarrow \alpha A + \beta B \in V \Rightarrow V$ is a vector space

$$A = \begin{bmatrix} 0 & a_1 & a_2 \\ -a_1 & 0 & a_3 \\ -a_2 & -a_3 & 0 \end{bmatrix} \quad \forall a_1, a_2, a_3 \in \mathbb{R}$$

$$V = \text{span} \left\{ \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \right\}$$

$$\dim V = 3.$$