$$= \frac{1}{\sqrt{2\pi}} \left[\frac{e^{(k-i\omega)x}}{k-i\omega} \right]_{-\infty}^{0}$$

$$= \frac{1}{\sqrt{2\pi}\,(k-i\omega)}$$

7. $f(x) = x \quad 0 < x < a$

$\quad\;\; = 0 \quad \text{otherwise}$

Fourier transform is

$$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)\, e^{-i\omega x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{a} x\, e^{-i\omega x}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[x \frac{e^{-i\omega x}}{(-i\omega)} - 1 \left(\frac{e^{-i\omega x}}{(-i\omega)^2} \right) \right]_0^a$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{a e^{-i\omega a}}{-i\omega} - \frac{e^{-i\omega a}}{i^2\omega^2} + \frac{1}{i^2\omega^2} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{i a e^{-i\omega a}}{\omega} + \frac{e^{-i\omega a}}{\omega^2} - \frac{1}{\omega^2} \right]$$

$$= \frac{e^{-i\omega a}(1 + ia\omega) - 1}{\sqrt{2\pi}\,\omega^2}$$

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$\therefore \quad c = \pm 3/\omega$

If we take c on the other side

ie $\quad c^2 u_t = u_{xx}$

then $c = \pm \omega/3$

It satisfies the equation

10. $u = e^x \cos y, \; e^x \sin y$

The Laplace equation is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

ie $\quad u_{xx} + u_{yy} = 0$

For the given function $u = e^x \cos y$

$u_x = e^x \cos y. \qquad u_y = -e^x \sin y$

$u_{xx} = e^x \cos y \qquad u_{yy} = -e^x \cos y$

Substituting in the Laplace eq^n

$$e^x \cos y - e^x \cos y = 0$$

$\therefore \quad u = e^x \cos y$ is a solution

Take $\quad u = e^x \sin y$

$u_x = e^x \sin y \qquad u_y = e^x \cos y$

$u_{xx} = e^x \sin y \qquad u_{yy} = -e^x \sin y$

Substituting in the Laplace equation

$$e^x \sin y - e^x \sin y = 0$$

$\therefore \quad u = e^x \sin y$ is a solution 10/10

19. $u_x + y^2 u = 0$

To find $u = u(x,y)$ of the above PDE

Since no x-derivatives occur it is of the form

$$\frac{du}{dy} + y^2 u = 0$$

$$\therefore \frac{du}{u} = -y^2\,dy$$

Integrating

$$\int \frac{du}{u} = -\int y^2\,dy$$

$$\ln u = -\frac{y^3}{3} + \log c$$

$$u = Ce^{-y^3/3}$$

where C is a constant

$$C = f(x)$$

$$\therefore \underline{u = c(x)e^{-y^3/3}} = f(x,y)$$

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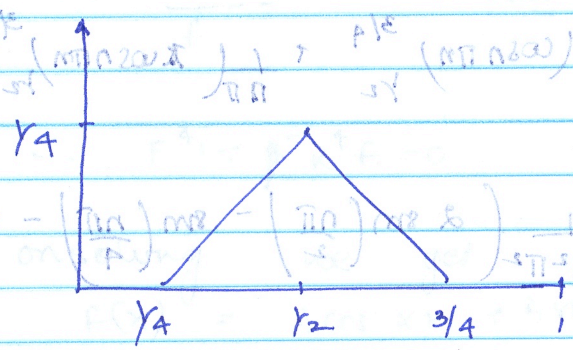

Given: $L = 1$

$u_t(x,0) = 0 \qquad u(x,0) = f(x)$

take

$$\ddot{G} + c^2\alpha^4 G = 0$$

the solution given by

$$G(t) = a\cos c\alpha^2 t + b\sin c\alpha^2 t$$

$a, b \Rightarrow$ constants

16 given $\dfrac{\partial^2 u}{\partial t^2} = -c^2 \dfrac{\partial^4 u}{\partial x^4}$

Boundary conditions

$u(0,t) = 0 \qquad u(L,t) = 0$

$u_{xx}(0,t) = 0 \qquad u_{xx}(L,t) = 0$

Initial conditions $u_t(x,0) = 0$

Let $u(x,t) = F(x)\,G(t)$ be the solution

$\therefore \dfrac{\partial^2 u}{\partial t^2} = F\ddot{G} \qquad \dfrac{\partial^4 u}{\partial x^4} = F^{(4)} G$

$\therefore F\ddot{G} = c^2 F^{(4)} G$

$\therefore \dfrac{-\ddot{G}}{c^2 G} = \dfrac{F^{(4)}}{F} = \alpha^4$

$F^{(4)} - \alpha^4 F = 0$

and $\ddot{G} + c^2\alpha^4 G = 0$

Using b.c

$u(0,t) = 0 \Rightarrow F(0)\,G(t) = 0 \Rightarrow F(0) = 0$

$G(t) = 0$ gives $u = 0$

$u(L,t) = 0 \Rightarrow F(L)\,G(t) = 0 \Rightarrow F(L) = 0$

$u_{xx}(L,t) = 0 \Rightarrow F^2(L)\,G(t) = 0 \Rightarrow F^2(L) = 0$

Solution of $F^{(4)} - \alpha^4 F = 0$ is

$F(x) = A\cos\alpha x + B\sin\alpha x + C\cosh\alpha x + D\sinh\alpha x.$

using $F(0) = 0$ we get

$0 = A + C$

$F(L) = 0 \Rightarrow 0 = A\cos\alpha L + B\sin\alpha L + C\cosh\alpha L + D\sinh\alpha L$

$F^2(x) = \alpha^2(-A\cos\alpha x - B\sin\alpha x + C\cosh\alpha x + D\sinh\alpha x)$

$F^2(0) = 0$

$\Rightarrow \quad 0 = B^2(-A + C) = -A + C = 0$

$\therefore \quad A = C = 0$

$F^2(L) = 0 \Rightarrow 0 = \alpha^2(-B\sin\alpha L + D\sinh\alpha L)$

$F(L) = 0 \Rightarrow 0 = B\sin\alpha L + D\sin\alpha L$

$\therefore \quad 2D\sinh\alpha L = 0 \Rightarrow D = 0$

$\sin\alpha L = 0$

$\therefore \quad \alpha L = n\pi$

$\therefore \quad \alpha = \frac{n\pi}{L} \qquad n = 1, 2, 3 \ldots$

$\therefore \quad F_n(x) = B_n \sin\frac{n\pi x}{L}$

Solution of $\ddot{G} + c^2\alpha^4 G = 0$ is

$G(t) = a\cos c\alpha^2 t + b\sin c\beta^2 t$

$G(t) = a_n\cos c\frac{n^2\pi^2}{L^2}t + b_n\sin c\frac{n^2\pi^2}{L^2}t$

$\therefore$

$$u(x,t) = \sum_{n=1}^{\infty} \left(B_n \sin \frac{n\pi x}{L}\right)\left(a_n \cos c \frac{n\pi x}{L} + b_n \sin c \frac{n^2\pi^2 t}{L^2}\right)$$

$$= \sum_{n=1}^{\infty} E_n \sin\left(\frac{n\pi x}{L}\right) \cos\left(\frac{n^2\pi^2}{L}\right) t$$

Initial vel. ie $u_t(x,0) = 0 \Rightarrow b_n = 0$

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