12.4

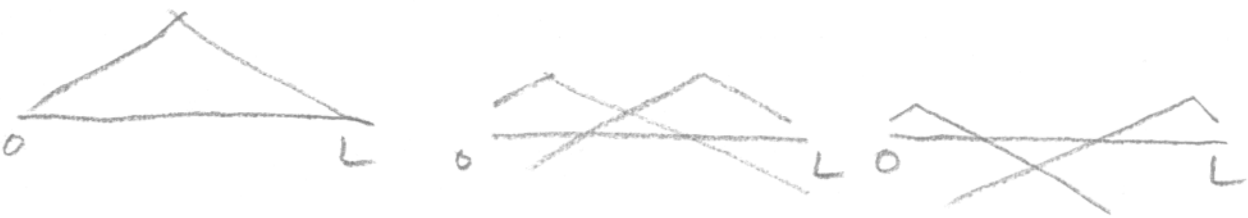
(8) pg. 556 $f(x) = kx(1-x)$ $L=1$, $c=1$, $k=.01$, ends fixed, 0 initial velocity ($u_t(x,0)=0$)

$u(x,t) = \frac{1}{2}[f(x+ct)+f(x-ct)]$

$\Rightarrow u(x,t) = \frac{k}{2}\left[(x+t)(1-(x+t)) + (x-t)(1-(x-t))\right] = \frac{k}{2}\left[(x+t)(1-x-t)+(x-t)(1-x+t)\right] =$

$= \frac{k}{2}\left[x+\cancel{t}-x^2-\cancel{xt}-\cancel{tx}-t^2+x-\cancel{t}-x^2+\cancel{xt}+\cancel{tx}-t^2\right] = \frac{k}{2}\left[2x-2x^2-2t^2\right] =$

$= \underline{k(x-x^2-t^2)}$



(19) pg. 556 $\underline{u_{tt} = c^2 u_{xx}}$, $c^2 = \frac{E}{\rho}$

Elastic rod:
fastened @ one end $-\ u(0,t)=0$ BCs
free @ other end $u_x(L,t)=0$
$u(x,0)=f(x)$ ICs
$u_t(x,0)=0$ zero initial vel.

Show: $u = \sum_{n=0}^{\infty} A_n \sin p_n x \cos p_n ct$; $A_n = \frac{2}{L}\int_0^L f(x)\sin p_n x\, dx$

$p_n = \frac{(2n+1)\pi}{2L}$

$u(x,t) = F(x)G(t) \Rightarrow u_{tt} = F\ddot{G}$, $u_{xx} = F''G \Rightarrow F\ddot{G} = c^2F''G \ / \div FGc^2$

$\Rightarrow \frac{\ddot{G}}{c^2G} = \frac{F''}{F} = \text{cons.} = k \Rightarrow F''-kF=0 \ \& \ \ddot{G}-kc^2G=0$

BCs

$u(x,t) = F(x)G(t)$; for $u(0,t)=0=F(0)G(t)$, $u_x(L,t)=0=F'(L)G(t)$ for $t \geq 0$; $G(t) \neq 0$ since if $G(t)=0 \Rightarrow u(x,t)\equiv 0$ violates $u(x,0)=f(x)$

- let $k=0 \Rightarrow F''-kF=0 \Rightarrow F''=0 \Rightarrow F=ax+b$ & $F'=a=0$ for $x=L$; $F=0=a(0)+b \Rightarrow b=0$ for $x=0$ $\Rightarrow F\equiv 0$ no interest
- let $k>0 \Rightarrow F''=kF \Rightarrow s^2=k \Rightarrow s=\pm\sqrt{k} \Rightarrow F = A_1e^{\sqrt{k}x}+B_1e^{-\sqrt{k}x}$

$F(0)=0=A_1+B_1 \Rightarrow A_1=-B_1$;
$F' = \sqrt{k}A_1e^{\sqrt{k}x} - \sqrt{k}B_1e^{-\sqrt{k}x} \Rightarrow F(L)=0=\cancel{\sqrt{k}}A_1e^{L\sqrt{k}} - \cancel{\sqrt{k}}B_1e^{-L\sqrt{k}} \Rightarrow 0 = -B_1e^{L\sqrt{k}} - B_1e^{-L\sqrt{k}} \Rightarrow F\equiv 0$

$\Rightarrow k$ must be <0, let $\underline{k=-p^2}$

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12.4

(19) pg. 556 $\Rightarrow F'' + p^2 F = 0 \Rightarrow s^2 = -p^2 \Rightarrow s = \pm ip$

$\Rightarrow F = A\cos px + B\sin px \; ; \; F(0) = 0 = A$ otherwise $F \equiv 0$

$F' = -Ap\sin px + Bp\cos px \; , \; F'(L) = 0 = Bp\cos pL$ $B \neq 0 \Rightarrow \cos pL = 0$

$\Rightarrow pL = \frac{(2n+1)\pi}{2} \Rightarrow \boxed{p = \frac{(2n+1)\pi}{2L} \Rightarrow F_n(x) = \sin p_n x \text{ for } B = 1}$

eigen value   eigen function

$\ddot{G} - kc^2 G = 0$ Let $\lambda_n = cp_n$, $k = -p^2$ $\Rightarrow \lambda_n^2 = c^2 p_n^2 \Rightarrow \ddot{G} = -\lambda_n^2 = \pm i\lambda_n$

$\Rightarrow G_n(t) = A_n\cos\lambda_n t + A_n^*\sin\lambda_n t$

$$\Rightarrow \boxed{u(x,t) = \sum_{n=1}^{\infty}(A_n\cos\lambda_n t + A_n^*\sin\lambda_n t)\sin p_n x}$$

ICs $u(x,0) = f(x) = \sum_{n=1}^{\infty} A_n \sin p_n x \Rightarrow \boxed{A_n = \frac{2}{L}\int_0^L f(x)\sin p_n x\,dx}$

$u_t(x,t) = \sum_{n=1}^{\infty}(-\lambda_n A_n \sin\lambda_n t + \lambda_n A_n^*\cos\lambda_n t)\sin p_n x$

$\Rightarrow u_t(x,0) = \sum_{n=1}^{\infty} A_n^*\lambda_n \sin p_n x = 0, \; \lambda_n = cp_n$

$A_n^* = \frac{4}{(2n+1)c\pi}\int_0^L g(x)\sin p_n x\,dx$

zero initial vel. ($u_t(x,0) = 0$) $\Rightarrow A_n^* = 0$
$g(x) = 0$

$$\Rightarrow \boxed{u(x,t) = \sum_{n=1}^{\infty} A_n\cos\lambda_n t\,\sin p_n x}$$

12.6

7. pg. 566

find temp. $u(x,t)$, silver bar (perf. insulated laterally)

$L = 10\,cm$ length

$\rho = 10.6\ g/cm^3$ density

$A = 1\,cm^2$, const. cross-sectional area $\qquad c^2 = \frac{K}{\sigma\rho}\ \frac{cm^2}{sec}$

$\sigma = .056\ cal/(g\,{}^\circ C)$ specific heat

$K = 1.04\ cal/(cm\cdot sec\cdot{}^\circ C)$ thermal conductivity

ends kept @ temp $0^\circ C$, initial temp $f(x)\,{}^\circ C$

$f(x) = x(10-x) = u(x,0)$ initial temp

heat eqn.: $u_t = c^2 u_{xx}$

$$u(x,t) = \sum_{n=1}^{\infty} u_n(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L}\, e^{-\lambda_n^2 t} \qquad \lambda_n = \frac{cn\pi}{L}$$

$$\Rightarrow u(x,0) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{10} = f(x) = x(10-x)$$

$$B_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx = .2\int_0^{10}(10x-x^2)\sin\frac{n\pi x}{10}\,dx =$$

$$= .2\int_0^{10} 10x\sin\frac{n\pi x}{10}\,dx - .2\int_0^{10} x^2\sin\frac{n\pi x}{10}\,dx =$$

$u = x \quad dv = \sin\frac{n\pi x}{10}dx$

$du = dx \quad v = -\frac{10}{n\pi}\cos\frac{n\pi x}{10}$

$$= 2\left[-\frac{10x}{n\pi}\cos\frac{n\pi x}{10}\Big|_0^{10} + \frac{10}{n\pi}\int_0^{10}\cos\frac{n\pi x}{10}\,dx\right] -$$

$u_1 = x^2 \quad dv_1 = dv$

$du_1 = 2x\,dx \quad v_1 = v$

$$- .2\left[-\frac{x^2 10}{n\pi}\cos\frac{n\pi x}{10}\Big|_0^{10} + \frac{20}{n\pi}\int_0^{10} x\cos\frac{n\pi x}{10}\,dx\right] =$$

$\sin 0 = 0$; $\sin n\pi = 0$, $n = 1,2,\ldots$

$\cos n\pi = (-1)^n$

$$= 2\left[-\frac{100}{n\pi}\cos n\pi + \frac{100}{n^2\pi^2}\cancel{\sin\frac{n\pi x}{10}}\Big|_0^{10}\right] - .2\left[-\frac{1000}{n\pi}\cos n\pi + \frac{20}{n\pi}\left(\frac{10}{n\pi}x\sin\frac{n\pi x}{10}\Big|_0^{10} - \frac{10}{n\pi}\int_0^{10}\sin\frac{n\pi x}{10}dx\right)\right.$$

$$= \frac{-200}{n\pi}\cancel{\cos n\pi} + \frac{200}{n\pi}\cancel{\cos n\pi} - \frac{.2(20)}{n\pi}\left[\frac{100}{n\pi}\cancel{\sin n\pi} + \right.$$

$u_2 = x \quad dv_2 = \cos\frac{n\pi x}{10}dx$

$du_2 = dx \quad v_2 = \frac{10}{n\pi}\sin\frac{n\pi x}{10}$

$$\left. + \frac{100}{n^2\pi^2}\cos\frac{n\pi x}{10}\Big|_0^{10}\right] = -\frac{400}{n^3\pi^3}(\cos n\pi - 1) = \boxed{\frac{400}{n^3\pi^3}(1-(-1)^n)}$$

$$\lambda_n = \frac{cn\pi}{L} \Rightarrow \lambda_n^2 = c^2\frac{n^2\pi^2}{L^2} = \frac{K}{\sigma\rho}\frac{\pi^2}{L^2}n^2 = \frac{1.04}{.056\cdot 10.6}\frac{\pi^2}{10^2}n^2\ \frac{\cancel{cal}}{\cancel{cm}\,sec\,\cancel{{}^\circ C}}\ \frac{\cancel{g}\,\cancel{{}^\circ C}}{\cancel{cal}}\ \frac{\cancel{cm^3}}{\cancel{g}}\ \frac{}{\cancel{cm^2}} =$$

$$= \boxed{.172918\,n^2\,sec^{-1}}$$

$$\Rightarrow \boxed{u(x,t) = \frac{400}{\pi^3}\sum_{n=1}^{\infty}\frac{(1-(-1)^n)}{n^3}\sin\frac{n\pi x}{10}\,e^{-.172918n^2t}} \quad \text{for odd } n$$

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12.6

(10). Assume ends of the bar were kept @ $100°C$

pg. 567 @ $t=0$, $u(L,t) = 0\ °C$ find $u(x,t)$ @ $x = L/2$
$u(0,t) = 100\ °C$ $t = 1, 2, 3, 10, 50$ sec
$u(x,0) = 100\ °C$

$$u_t = c^2 u_{xx}$$

let $u(x,t) = (0-100)\frac{x}{L} + 100 + w(x,t) = 100 - 100\frac{x}{L} + w(x,t)$

$\Rightarrow u(0,t) = 100°C = 100 + w(0,t) \Rightarrow \underline{w(0,t) = 0°C}$

$u(L,t) = 0°C = \cancel{100} - \cancel{100} + w(L,t) \Rightarrow \underline{w(L,t) = 0°C}$

$\cancel{100} = u(x,0) = \cancel{100} - 100\frac{x}{L} + w(x,0) \Rightarrow \underline{w(x,0) = 100\frac{x}{L}}$

$w_t = c^2 w_{xx}$; $w(x,t) = \sum_{n=1}^{\infty} B_n \sin\frac{n\pi x}{L} e^{-\lambda_n^2 t}$; $\lambda_n = \frac{cn\pi}{L}$

$$B_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx$$

$f(x) = w(x,0) = 100\frac{x}{L}$

$u = x$ $dv = \sin\frac{n\pi x}{L}dx$
$du = dx$ $v = -\frac{L}{n\pi}\cos\frac{n\pi x}{L}$

$\Rightarrow B_n = \frac{2}{L}\left(\frac{100}{L}\right)\int_0^L x\sin\frac{n\pi x}{L}\,dx =$

$$= \frac{200}{L^2}\left[-\frac{Lx}{n\pi}\cos\frac{n\pi x}{L}\Big|_0^L + \frac{L}{n\pi}\int_0^L \cos\frac{n\pi x}{L}\,dx\right] = \frac{200}{L^2}\left[\frac{-L^2}{n\pi}\cos n\pi + \frac{L^2}{n^2\pi^2}\underbrace{\sin\frac{n\pi x}{L}\Big|_0^L}_{0}\right] =$$

$$= \boxed{\frac{-200}{n\pi}(-1)^n} \Rightarrow \boxed{w(x,t) = -\frac{200}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{L} e^{-\left(\frac{cn\pi}{L}\right)^2 t}}$$

$$\Rightarrow \boxed{u(x,t) = \frac{100}{L}(L-x) - \frac{200}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\sin\frac{n\pi x}{L} e^{-\left(\frac{cn\pi}{L}\right)^2 t}}$$

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12.6

(11) pg. 567 completely insulated bar : $u_x(0,t)=0$, $u_x(L,t)=0$ BCs, $u(x,t)=f(x)$

$u_t=c^2u_{xx}$ heat eqn. $\Rightarrow u(x,t)=F(x)\cdot G(t) \Rightarrow u_t=F\dot{G}$, $u_{xx}=F''G$

$\Rightarrow F\dot{G}=c^2F''G \,/\div FGc^2 \Rightarrow \frac{\dot{G}}{c^2G}=\frac{F''}{F}=\text{cons.}=-p^2$ const. $\neq 0$ or <0 since then $u\equiv 0$ ref. 19 on 11/9/82/9

- $\Rightarrow F''+p^2F=0$
$\Rightarrow s^2=-p^2 \Rightarrow s=\pm ip \Rightarrow \boxed{F(x)=A\cos px+B\sin px}$

BCs: $u_x(0,t)=0=F'(0)\,G(t)$
$u_x(L,t)=0=F'(L)\,G(t)$ $G(t)\neq 0$ since then $u\equiv 0$

$\Rightarrow F'(x)=-Ap\sin px+Bp\cos px \Rightarrow F'(0)=0=Bp$ $p\neq 0 \Rightarrow \underline{B=0}$

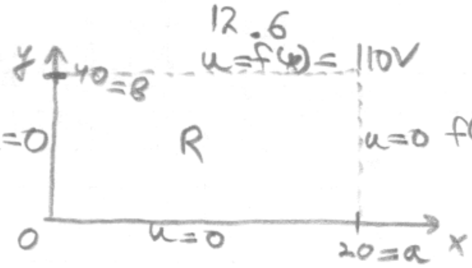
$F'(L)=0=-Ap\sin pL$ $A\neq 0$ or $u\equiv 0$ $\Rightarrow \sin pL=0 \Rightarrow pL=n\pi$
$\Rightarrow p=\frac{n\pi}{L}$, $n=1,2,3,\ldots$

For $A=1 \Rightarrow \boxed{F_n(x)=\cos\frac{n\pi}{L}x}$

- $\dot{G}+p^2c^2G=0$ Let $\lambda_n^2=c^2p_n^2=c^2\left(\frac{n\pi}{L}\right)^2 \Rightarrow \dot{G}+\lambda_n^2G=0$
$\Rightarrow s+\lambda_n^2=0 \Rightarrow s=-\lambda_n^2 \Rightarrow \boxed{G_n(t)=B_ne^{-\lambda_n^2t}}$ B_n is a const.
$\Rightarrow u_n(x,t)=\left(\sum_{n=1}^{\infty}B_n\cos\frac{n\pi}{L}x\,e^{-\lambda_n^2t}\right)+A_0$ (cos is an even func.)

12.6

(18) 567 pg.



$0 \le x \le 20$
$0 \le y \le 40$

f(x), potential is 110 V (upper side)
other sides are grounded

find the potential in the rectangle $u(x,y)$

from (17): $u(x,y) = \sum_{n=1}^{\infty} A_n^* \sin \frac{n\pi x}{a} \sinh \frac{n\pi y}{a}$ $\quad n = 1,2,3,\ldots$

(18): $A_n^* = \frac{2}{a \sinh(n\pi b/a)} \int_0^a f(x) \sin \frac{n\pi x}{a}\, dx =$

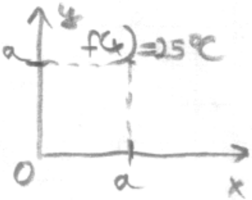
$= \frac{\cancel{2}^{\,1}}{\cancel{20}_{10} \sinh(2n\pi)} \int_0^{20} 110 \sin \frac{n\pi x}{20}\, dx = -\frac{110}{1\cancel{0}\sinh(2n\pi)} \frac{2\cancel{0}}{n\pi} \cos \frac{n\pi x}{20} \Big|_0^{20} =$

$= -\frac{220}{n\pi \sinh(2n\pi)} (\cos n\pi - 1) = \boxed{\frac{220}{n\pi \sinh(2n\pi)} (1 - (-1)^n)}$

$\Rightarrow$ if n is even: $A_n^* = 0$
n is odd: $A_n^* = \frac{440}{n\pi \sinh(2n\pi)}$

$$\Rightarrow \boxed{u(x,y) = \frac{440}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)\sinh(2\pi(2n-1))} \sin \frac{(2n-1)\pi x}{20} \sinh \frac{(2n-1)\pi y}{20}}$$

(21) pg. 567



$a = 24$ find $u(x,y)$ SS temp.
upper side is @ 25°C, other sides @ 0°C $= f(x) = 25$
the faces are perf. insulated

$u(x,y) = \sum_{n=1}^{\infty} A_n^* \sin \frac{n\pi x}{24} \sinh \frac{n\pi y}{24}$; $A_n^* = \frac{-2 \cdot 25}{\cancel{24} \sinh(n\pi)} \frac{\cancel{24}}{n\pi} \cos \frac{n\pi x}{24} \Big|_0^{24} =$

$= \frac{50}{n\pi \sinh(n\pi)} (1 - (-1)^n) \Rightarrow A_n^* = 0$ if n is even and $A_n^* = \frac{100}{n\pi \sinh(n\pi)}$ if n is odd

$$\Rightarrow \boxed{u(x,y) = \frac{100}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)\sinh((2n-1)\pi)} \sin \frac{(2n-1)\pi x}{24} \sinh \frac{(2n-1)\pi y}{24}}$$

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12.10

$r = \sqrt{x^2+y^2}$, $\tan\theta = \frac{y}{x}$

(9)

pg. 591 ③ Show that $u_n = r^n \cos n\theta$, $u_n = r^n \sin n\theta$, $n = 0, 1, \dots$ are soln.

of Laplace's eqn. $\nabla^2 u = 0$ $\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} =$

$= u_{rr} + \frac{1}{r} u_r + \frac{1}{r^2} u_{\theta\theta}$

$u(r,\theta) = W(r) Z(\theta)$

$u_{rr} = W''$
$u_r = W'$
$u_{\theta\theta} = \ddot{Z}$

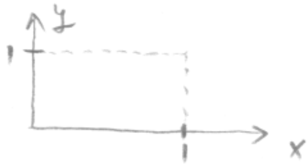
$\Rightarrow \overset{5}{W''} + \frac{1}{r} W' + \frac{1}{r^2}\ddot{Z} = 0 \Rightarrow \frac{rW''}{r} + \frac{1}{r} W' = -\frac{1}{r^2}\ddot{Z} = \text{cons.}$

$\Rightarrow \frac{rW''}{r} + \frac{1}{r} W' = -p^2$; $-\frac{1}{r^2}\ddot{Z} = -p^2$

12.9

(4) Repr. $f(x,y)$ by a series
pg. 584

$f(x,y)=1 \qquad a=b=1$



$$u(x,y,0)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}=f(x,y)$$

$$B_{mn}=\frac{4}{ab}\int_0^b\int_0^a f(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dx\,dy=$$

$$=4\int_0^1\sin m\pi x\,dx\int_0^1\sin n\pi y\,dy=4\underbrace{\left(-\frac{1}{m\pi}\right)\cos m\pi x\Big|_0^1}_{①}\underbrace{\left(-\frac{1}{n\pi}\right)\cos n\pi y\Big|_0^1}_{②}=$$

①: $-\frac{1}{m\pi}(\cos m\pi-1)=\frac{1}{m\pi}(1-(-1)^m)$ m odd

②: $-\frac{1}{n\pi}(\cos n\pi-1)=\frac{1}{n\pi}(1-(-1)^n)$ n odd

$\Rightarrow B_{mn}=\frac{4}{mn\pi^2}(2)(2)=\frac{16}{mn\pi^2}$ m, n both odd

$$\Rightarrow \boxed{u(x,y,0)=\frac{16}{\pi^2}\sum_{m,n,\,odd}^{\infty}\sum^{\infty}\frac{1}{mn}\sin m\pi x\sin n\pi y}$$

(5). $f(x,y)=y \quad a=b=1$

$$B_{mn}=4\int_0^1\int_0^1 y\sin m\pi x\sin n\pi y\,dx\,dy=4\underbrace{\int_0^1 1\sin m\pi x\,dx}_{①}\underbrace{\int_0^1 y\sin n\pi y\,dy}_{2}$$

① $\int_0^1\sin m\pi x\,dx=\frac{1}{m\pi}(2)=\boxed{\frac{2}{m\pi}}$ m odd

② $\int_0^1 y\sin n\pi y\,dy=\quad u=y \quad dv=\sin n\pi y\,dy \quad du=1\,dy \quad v=-\frac{1}{n\pi}\cos n\pi y$

$$=-\frac{y}{n\pi}\cos n\pi y\Big|_0^1+\frac{1}{n\pi}\int_0^1\cos n\pi y\,dy=-\frac{1}{n\pi}\cos n\pi+\frac{1}{(n\pi)^2}\sin n\pi y\Big|_0^1=\boxed{-\frac{1}{n\pi}(-1)^n}$$

$\Rightarrow B_{mn}=\frac{8}{m\pi}\left(-\frac{1}{n\pi}\right)(-1)^n$ m must be odd, $n=1,2,3,\ldots$

$$\Rightarrow \boxed{u(x,y,0)=\frac{8}{\pi^2}\sum_{m\,odd}^{\infty}\sum_{n=1}^{\infty}\frac{1}{m}\left(-\frac{1}{n}\right)(-1)^n\sin m\pi x\sin n\pi y}$$

12.9

(4) $f(x,y) = xy$ a, b are arbitrary

pg. 584

$$B_{mn} = \frac{4}{ab}\int_0^b\int_0^a f(x,y)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\,dx\,dy =$$

$$= \frac{4}{ab}\underbrace{\int_0^a x\sin\frac{m\pi x}{a}\,dx}_{①}\ \underbrace{\int_0^b y\sin\frac{n\pi y}{b}\,dy}_{②}$$

$u = x$ $\quad dv = \sin\frac{m\pi x}{a}\,dx$

$du = 1\,dx$ $\quad v = \frac{-a}{m\pi}\cos\frac{m\pi x}{a}$

①: $\int_0^a x\sin\frac{m\pi x}{a}\,dx = -\frac{xa}{m\pi}\cos\frac{m\pi x}{a}\Big|_0^a + \frac{a}{m\pi}\int_0^a\cos\frac{m\pi x}{a}\,dx =$

$= -\frac{a^2}{m\pi}\cos m\pi + \left(\frac{a}{m\pi}\right)^2 \sin\frac{m\pi x}{a}\Big|_0^a = \boxed{-\frac{a^2(-1)^n}{m\pi}}$

②: $\int_0^b y\sin\frac{n\pi y}{b}\,dy = \boxed{\frac{-b^2(-1)^n}{n\pi}}$

$\Rightarrow B_{mn} = \frac{4}{\cancel{ab}}\left(\frac{a^{\cancel{2}}b^{\cancel{2}}}{mn\pi^2}\right)(-1)^m(-1)^n =$

$= \boxed{\frac{4ab(-1)^m(-1)^n}{mn\pi^2}}$

$\Rightarrow \boxed{u(x,y,0) = \frac{4}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{ab}{mn}(-1)^m(-1)^n\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}}$

(18). $A = ab \Rightarrow b = \frac{A}{a}$ $\quad m = n = 1$

pg. 585 $u_{mn}(x,y,t) = (B_{mn}\cos\lambda_{mn}t + B^*_{mn}\sin\lambda_{mn}t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$

$u_{11} = (B_{11}\cos\lambda_{11}t + B^*_{mn}\sin\lambda_{11}t)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$ diff. WRT a

$\lambda = \lambda_{mn} = c\pi\sqrt{\frac{m^2}{a^2}+\frac{n^2}{b^2}} \Rightarrow \lambda_{11} = c\pi\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} \Rightarrow \lambda_{11}^2 = c^2\pi^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$

$\Rightarrow \left(\frac{\lambda_{11}^2}{c^2\pi^2}\right)' = \left(\frac{1}{a^2}+\frac{1}{b^2}\right)' = \left(\frac{1}{a^2}+\frac{a^2}{A^2}\right)' = (a^{-2})' + \frac{1}{A^2}2a = -\frac{2}{a^3} + \frac{2a}{A^2}$ set to 0

$\Rightarrow -\frac{\cancel{2}}{a^3} + \frac{\cancel{2}a}{A^2} = 0 \Rightarrow \frac{a}{A^2} = \frac{1}{a^3} \Rightarrow a^4 = A^2 \Rightarrow \boxed{a^2 = A \quad \text{but } b = \frac{A}{a} = \frac{a^2}{a} = a}$