Mark Martin-Fanone

MA527 HW1

Due 8-29

P261 #9, 12, 13 P280 #3, 9, 18

P271 #12, 14, 17, 29 P287 #2, 9, 12, 14, 15, 17, 32, 34

7.1.9) $\vec{A} = \begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix}$ $\vec{B} = \begin{bmatrix} 0 & 5 & 2 \\ 5 & 3 & 4 \\ -2 & 4 & -2 \end{bmatrix}$ $\vec{u} = \begin{bmatrix} 1.5 \\ 0 \\ -3.0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix}$

$\vec{w} = \begin{bmatrix} -5 \\ -30 \\ 10 \end{bmatrix}$

$\vec{C} = \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}$ $\vec{D} = \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix}$ $\vec{E} = \begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix}$

$3\vec{A} = \begin{bmatrix} 0 & 6 & 12 \\ 18 & 15 & 15 \\ 3 & 0 & -9 \end{bmatrix}$ $0.5\vec{B} = \begin{bmatrix} 0 & 2.5 & 1 \\ 2.5 & 1.5 & 2 \\ -1 & 2 & -1 \end{bmatrix}$

$3\vec{A} + 0.5\vec{B} = \begin{bmatrix} 0 & 8.5 & 13 \\ 20.5 & 16.5 & 17 \\ 2 & 2 & -10 \end{bmatrix}$

$3\vec{A} + 0.5\vec{B} + \vec{C}$ is not defined because $\vec{C}$ is not the same size as $\vec{A}$ and $\vec{B}$

7.1.12) $(\vec{C} + \vec{D}) + \vec{E} = \begin{bmatrix} 1 & 3 \\ 3 & 4 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 6 & 8 \\ 6 & -2 \end{bmatrix}$

$(\vec{D} + \vec{E}) + \vec{C}$ = same by rules 3a & b

$0(\vec{C} - \vec{E}) + 4\vec{D} = = \vec{0} + 4\vec{D} = \begin{bmatrix} -16 & 4 \\ 20 & 0 \\ 8 & -4 \end{bmatrix}$ rules 3a, c, 4a

$\vec{A} - 0\vec{C}$ is not defined because $\vec{C}$ is not the same size as $\vec{A}$
so $0\vec{C}$ is not the same size as $\vec{A}$

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7.1.13) $(2 \cdot 7)\vec{C} = 14\vec{C} = \begin{bmatrix} 70 & 28 \\ -28 & 56 \\ 14 & 0 \end{bmatrix}$

$2(7\vec{C}) =$ same by rule 4c

$-\vec{D} + 0\vec{E} = -\vec{D} + \vec{0} = \begin{bmatrix} 4 & -1 \\ -5 & 0 \\ -2 & 1 \end{bmatrix} = -\vec{D}$

$\vec{E} - \vec{D} + \vec{C} + \vec{u}$ is not defined because $\vec{u}$ is not the same size as $\vec{E}$, $\vec{D}$, and $\vec{C}$

7.2.12)

$\vec{A} = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \qquad \vec{B} = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

$\vec{C} = \begin{bmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{bmatrix} \qquad a = [1 \quad -2 \quad 0] \qquad b = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$

$\vec{A}\,\vec{A}^T = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & -2 & 1 \\ -2 & 1 & 2 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 16+4+9 & -8-2+18 & 4-4+6 \\ -8-2+18 & 4+1+36 & -2+2+12 \\ 4-4+6 & -2+2+12 & 1+4+4 \end{bmatrix}$

$= \begin{bmatrix} 29 & 8 & 6 \\ 8 & 41 & 12 \\ 6 & 12 & 9 \end{bmatrix}$

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$$\vec{A}^2 = \vec{A}\vec{A} = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 16+4+3 & -8-2+6 & 12-12+6 \\ -8-2+6 & +4+1+12 & -6+6+12 \\ 4-4+2 & -2+2+4 & 3+12+4 \end{bmatrix}$$

$$= \begin{bmatrix} 23 & -4 & 6 \\ -4 & 17 & 12 \\ 2 & 4 & 19 \end{bmatrix}$$

$$\vec{B}\vec{B}^T = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1+9 & -3-3 & 0 \\ -3-3 & 9+1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\vec{B}^2 = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

7.2, 14)

$$3\vec{A} - 2\vec{B} = 3\begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} - 2\begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & -6 & 9 \\ -6 & 3 & 18 \\ 3 & 6 & 6 \end{bmatrix} - \begin{bmatrix} 2 & -6 & 0 \\ -6 & 2 & 0 \\ 0 & 0 & -4 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 9 \\ 0 & 1 & 18 \\ 3 & 6 & 10 \end{bmatrix}$$

$$(3\vec{A} - 2\vec{B})^T = \begin{bmatrix} 10 & 0 & 9 \\ 0 & 1 & 18 \\ 3 & 6 & 2 \end{bmatrix}^T = \begin{bmatrix} 10 & 0 & 3 \\ 0 & 1 & 6 \\ 9 & 18 & 10 \end{bmatrix}$$

$$3\vec{A}^T - 2\vec{B}^T = (3\vec{A} - 2\vec{B})^T = \begin{bmatrix} 10 & 0 & 3 \\ 0 & 1 & 6 \\ 9 & 18 & 10 \end{bmatrix}$$

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$$(3\vec{A}-2\vec{B})^T\vec{a}^T = \begin{bmatrix} 10 & 0 & 3 \\ 0 & 1 & 6 \\ 9 & 18 & 10 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ 9-36 \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -27 \end{bmatrix}$$

7.2.17)

$$\vec{A}\vec{B}\vec{C} = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 4+6 & -12-2 & -6 \\ -2-3 & +6+1 & -12 \\ 1-6 & -3+2 & -4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -42+12 & 10-28 \\ +21+24 & -5+14 \\ -3+8 & -5-2 \end{bmatrix} = \begin{bmatrix} -30 & -18 \\ 45 & 9 \\ 5 & -7 \end{bmatrix}$$

$\vec{A}\vec{B}\vec{a}$ is not defined because the number of columns of $\vec{A}\vec{B}$ (three) do not equal the number of rows of $\vec{a}$ (one)

$$\vec{A}\vec{B}\vec{b} = \begin{bmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 30-14+6 \\ -15+7+12 \\ -15-1+4 \end{bmatrix} = \begin{bmatrix} 22 \\ 4 \\ -12 \end{bmatrix}$$

$\vec{C}\vec{a}^T$ is not defined ($\vec{C}$ has 2 columns; $\vec{a}^T$ has 3 rows)

7.2.29)

$$\vec{p} = \begin{bmatrix} 35 \\ 62 \\ 30 \end{bmatrix};\quad \vec{v} = \vec{A}\vec{p} = \begin{bmatrix} 400 & 60 & 240 \\ 100 & 120 & 500 \end{bmatrix}\begin{bmatrix} 35 \\ 62 \\ 30 \end{bmatrix} = \begin{bmatrix} 14000+3720+7200 \\ 3500+7440+15000 \end{bmatrix}$$

$$= \begin{bmatrix} 24920 \\ 25940 \end{bmatrix}$$

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7.3.3)

$$x + y - z = 9$$
$$8y + 6z = -6$$
$$-2x + 4y - 6z = 40$$

$$2 \begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ -2 & 4 & -6 & 40 \end{bmatrix}$$

$$-\frac{6}{8} \begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 6 & -8 & 58 \end{bmatrix} \qquad \begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 0 & -\frac{25}{2} & \frac{125}{2} \end{bmatrix}$$

$$-\frac{25}{2} z = \frac{125}{2} \qquad z = -5$$

$$8y + 6z = -6 \qquad y = 3$$

$$x + y - z = 9 \qquad x = 1$$

7.3.9)

$$-2y - 2z = -8$$
$$3x + 4y - 5z = 13$$

$$\begin{bmatrix} 0 & -2 & -2 & -8 \\ 3 & 4 & -5 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 4 & -5 & 13 \\ 0 & -2 & -2 & -8 \end{bmatrix}$$

$$y = \frac{2z - 8}{-2} = -z + 4$$

$$3x + 4(-z + 4) - 5z = 13$$

$$3x + -4z + 16 - 5z = 13$$

$$3x = -3 + 9z$$

$$x = -1 + 3z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} z$$

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7.3.18)

$$-I_1 + I_2 + I_3 = 0$$
$$I_1 - I_2 - I_3 = 0$$
$$4I_1 + 12I_2 = 36$$
$$12I_2 + 8I_3 = 24$$

$$\begin{bmatrix} -1 & 1 & 1 & 0 \\ 1 & -1 & -1 & 0 \\ 4 & 12 & 0 & 36 \\ 0 & 12 & -8 & 24 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 & 0 \\ 0 & 12 & -8 & 24 \\ 0 & 16 & 4 & 36 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 1 & -2/3 & 2 \\ 0 & 0 & 44/3 & 4 \end{bmatrix}$$

$I_1 = 2.4545$
$I_2 = 2.1818$
$I_3 = 0.27273$

7.4.2)

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} r = 2: \quad [a, b]; \; [0, a - \frac{b^2}{a}]$$

column space is the same

7.4.9)

$$\begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 9 & 8 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix} r = 3: \quad \begin{matrix} [9\ 0\ 1\ 0] \\ [0\ 0\ 1\ 0] \\ [0\ 9\ 8\ 9] \end{matrix}$$

column space is the same

7.4.12)

$B^T A^T = (AB)^T$ (rule 10d §7.2)

rank AB = rank $(AB)^T$ (thm 3 §7.4)

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7.4.14)

A is an $n \times m$ matrix where $n \neq m$ (non-square)

if $m < m$; the rows are linearly dependent by thm 4.

if $m > n$ then A^T is $m \times n$ where $n < M$

$\therefore$ the rows of A^T are linearly dependent so the columns of A are linearly dependent

7.4.15) if An $n \times n$ matrix has rank $p = n$ (ie the n. row vectors are linear independent) then it has $p = n$ independent column vectors. by thm 3.

if m nxn matrix

7.4.17)

$$\begin{bmatrix} 3 & 4 & 0 & 2 \\ 2 & -1 & 3 & 7 \\ 1 & 16 & -12 & -22 \end{bmatrix} : \begin{bmatrix} 1 & 16 & -12 & -22 \\ 0 & -44 & +36 & +68 \\ 0 & -33 & 27 & 51 \end{bmatrix} = \begin{bmatrix} 1 & 16 & -12 & -22 \\ 0 & 11 & -9 & -17 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

the set is not linearly independent

7.4.32) $3v_1 - 2v_2 + v_3 = 0$

$4v_1 + 5v_2 = 0$

$r(A) = 2$

$\# col(A) = 3$

nullity $A = 1$

$A\vec{v} = \vec{0}$

$$\begin{bmatrix} 3 & -2 & 1 \\ 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-\frac{29}{4}$ 6 $\frac{33}{4}$

1 $-\frac{4}{3}$ $+\frac{19}{3}$

$$\begin{bmatrix} 1 & -2/3 & \frac{1}{3} \\ 0 & 23/3 & -\frac{4}{3} \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$v_3 = \frac{23}{4} v_2$

$1 - \frac{8}{12} + \frac{23}{12} = 0$

$v_1 = -\frac{15}{12} = -\frac{5}{4} v_2$

(continued)

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7. 4.32) continued:

$\{ v_2 [-\frac{5}{4} \quad 1 \quad \frac{23}{4}] \}$ is a vector space because it is the sol'n of the homogenous system $\vec{A}\vec{v} = \vec{0}$

the dimension is equal to the nullity of $\bar{A}$ dim = 1

basis $[-5 \quad 4 \quad 23]$

7.4.34 the set is not a vector space because the linear combination of any two elements of the set may not be in the set.

for example if $\alpha > 0, \beta = 0$ $\alpha\vec{a} + \beta\vec{b} = \alpha\vec{a}$ $(|a_j| = \alpha > 1$ for $j = 1..n)$ is not in the set.