1.1-9

$$\bar{A} = \begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix} \quad \bar{B} = \begin{bmatrix} 0 & 5 & 2 \\ 5 & 3 & 4 \\ -2 & 4 & -2 \end{bmatrix} \quad \bar{C} = \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix} \quad \bar{D} = \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix}$$

$$\bar{E} = \begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix}$$

$$3\bar{A} = \begin{bmatrix} 0 & 6 & 12 \\ 18 & 15 & 15 \\ 3 & 0 & -9 \end{bmatrix}$$ from $c\bar{A} = [ca_{jk}]$

$$0.5\bar{B} = \begin{bmatrix} 0 & 2.5 & 1 \\ 2.5 & 1.5 & 2 \\ -1 & 2 & -1 \end{bmatrix}$$ from $c\bar{A} = [ca_{jk}]$

$$3\bar{A} + 0.5\bar{B} = \begin{bmatrix} 0 & 8.5 & 13 \\ 20.5 & 16.5 & 17 \\ 2 & 2 & -10 \end{bmatrix}$$ from $\bar{A} + \bar{B} = [a_{jk} + b_{jk}]$ if $\bar{A}$ and $\bar{B}$ are same size

$3\bar{A} + 0.5\bar{B} + \bar{C}$ = undefined All matrices must have same size

7.1-12

$$(\bar{C} + \bar{D}) + \bar{E} = \begin{bmatrix} 1 & 3 \\ 3 & 4 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 6 & 8 \\ 6 & -2 \end{bmatrix}$$

demonstrates that matrix addition is associative

$$(\bar{D} + \bar{E}) + \bar{C} = \begin{bmatrix} -4 & 3 \\ 8 & 4 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 6 & 8 \\ 6 & -2 \end{bmatrix}$$

$$0(\bar{C} - \bar{E}) + 4\bar{D} = \begin{bmatrix} -16 & 4 \\ 20 & 0 \\ 8 & -4 \end{bmatrix}$$

$\bar{A} - \bar{0}\bar{C} =$ undefined   size A $\neq$ size $0\bar{C}$

$\boxed{7.1\text{-}13}$

$$(2\cdot 7)\,\bar{C} = 14\begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 70 & 28 \\ -28 & 56 \\ 14 & 0 \end{bmatrix}$$

demonstrates $c(k\bar{A}) = (ck)\bar{A}$

$$2(7\bar{C}) = 2\begin{bmatrix} 35 & 14 \\ -14 & 28 \\ 7 & 0 \end{bmatrix} = \begin{bmatrix} 70 & 28 \\ -28 & 56 \\ 14 & 0 \end{bmatrix}$$

$$-\bar{D} + 0\bar{E} = \begin{bmatrix} 4 & -1 \\ -5 & 0 \\ -2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -1 \\ -5 & 0 \\ -2 & 1 \end{bmatrix}$$

$\bar{E} - \bar{D} + \bar{C} + \bar{U}$ = undefined (size of $\bar{U}$ differs from other matrices)

$\boxed{1.2\text{-}12}$

$$\bar{A}\bar{A}^T = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & -2 & 1 \\ -2 & 1 & 2 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 29 & 8 & 6 \\ 8 & 41 & 12 \\ 6 & 12 & 9 \end{bmatrix}$$

Symmetric as expected

$$\bar{A}^2 = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 23 & -4 & 6 \\ -4 & 17 & 12 \\ 2 & 4 & 19 \end{bmatrix}$$

$$\bar{B}\bar{B}^T = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Symmetric as expected

$\Updownarrow$ Matrix is symmetric... answer is the same

$$\bar{B}^2 = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 10 & -6 & 0 \\ -6 & 10 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$\boxed{1.2-14}$ ✓

$$A = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$3\bar{A} - 2\bar{B} =$

$$\begin{bmatrix} 12 & -6 & 9 \\ -6 & 3 & 18 \\ 3 & 6 & 6 \end{bmatrix} + \begin{bmatrix} -2 & 6 & 0 \\ 6 & -2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 9 \\ 0 & 1 & 18 \\ 3 & 6 & 10 \end{bmatrix}$$

$\xleftarrow{\text{transposed}}$

$$(3\bar{A} - 2\bar{B})^T = \begin{bmatrix} 10 & 0 & 3 \\ 0 & 1 & 6 \\ 9 & 18 & 10 \end{bmatrix}$$

$\Downarrow$

$$3\bar{A}^T - 2\bar{B}^T = (3\bar{A} - 2\bar{B})^T = \begin{bmatrix} 10 & 0 & 3 \\ 0 & 1 & 6 \\ 9 & 18 & 10 \end{bmatrix}$$

$$(3\bar{A} - 2\bar{B})^T \bar{a}^T = \begin{bmatrix} 10 & 0 & 3 \\ 0 & 1 & 6 \\ 9 & 18 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \\ -27 \end{bmatrix}$$

1.2-11 ✓

$$\bar{A}\bar{B}\bar{C} = \begin{bmatrix} 4 & -2 & 3 \\ -2 & 1 & 6 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & -3 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -30 & -18 \\ 45 & 9 \\ 5 & -7 \end{bmatrix}$$

$\bar{A}\bar{B}\bar{a}$ = not defined. $\bar{A}\bar{B}$ is 3×3 while $\bar{a}$ is 1×3. Inner dimensions don't match.

$$\bar{A}\bar{B}\bar{b} = \begin{bmatrix} 10 & -14 & -6 \\ -5 & 7 & -12 \\ -5 & -1 & -4 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 22 \\ 4 \\ -12 \end{bmatrix}$$

$\bar{C}\bar{a}^T$ = undefined. $\bar{C}$ is 3×2 while $\bar{a}^T$ is 3×1.

$\boxed{1.2\text{-}29}$ ✓

$$\begin{array}{cc} & \begin{array}{ccc} S & C & T \end{array} \\ \bar{A} = & \begin{bmatrix} 400 & 60 & 240 \\ 100 & 120 & 500 \end{bmatrix} \end{array} \begin{array}{l} \leftarrow \text{store 1} \\ \leftarrow \text{store 2} \end{array}$$

let $\bar{p} = \begin{bmatrix} 35 \\ 62 \\ 30 \end{bmatrix}$ such that $\bar{V} = \bar{A}\bar{p}$

expanded...

$$\begin{bmatrix} V_{F1} \\ V_{F2} \end{bmatrix} = \begin{bmatrix} 400 & 60 & 240 \\ 100 & 120 & 500 \end{bmatrix} \begin{bmatrix} 35 \\ 62 \\ 30 \end{bmatrix}$$

$$= \begin{bmatrix} 24920 \\ 25940 \end{bmatrix}$$

$\boxed{1.3-3}$ ✓

$$
\begin{aligned}
x + y - z &= 9 \\
8y + 6z &= -6 \\
-2x + 4y - 6z &= 40
\end{aligned}
$$

Form augmented matrix

$$
\begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ -2 & 4 & -6 & 40 \end{bmatrix}
\sim
\begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 6 & -8 & 58 \end{bmatrix}
\sim
\begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 4 & 3 & -3 \\ 0 & 3 & -4 & 29 \end{bmatrix}
$$

$$
\sim
\begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 4 & 3 & -3 \\ 0 & 0 & -\frac{25}{4} & 31.25 \end{bmatrix}
\sim
\begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 4 & 3 & -3 \\ 0 & 0 & 1 & -5 \end{bmatrix}
\sim
\begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & \underset{1}{\cancel{4}} & 0 & \underset{3}{\cancel{12}} \\ 0 & 0 & 1 & -5 \end{bmatrix}
$$

$$
\sim
\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{bmatrix}
$$

$x = 1$

$y = 3$

$z = -5$

$\boxed{1.3-4}$ ✓ Form augmented matrix

$$\begin{bmatrix} 0 & -2 & -2 & -8 \\ 3 & 4 & -5 & 13 \end{bmatrix} \sim \begin{bmatrix} 3 & 4 & -5 & 13 \\ 0 & 1 & 1 & 4 \end{bmatrix}$$

let $z = t$ (arbitrary)

$y = 4 - t$

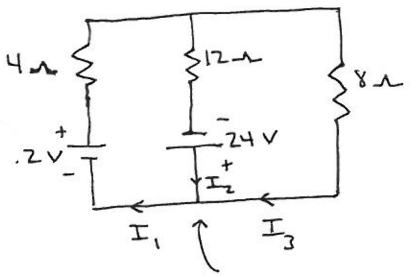
$x = 3t - 1$

$3x = 13 + 5t - 4(4-t)$

$= -3 + 9t$

$x = -1 + 3t$

$\boxed{1.3-18}$ ✓



$-I_1 + I_2 + I_3 = 0$

Loop 1

$$12 = 4I_1 + 12(I_1 - I_3) - 24$$
$$36 = 16I_1 - 12I_3$$

Loop 2

$$-24 = 8I_3 + 12(I_3 - I_1)$$
$$-24 = -12I_1 + 20I_3$$

$$\left[\begin{array}{cc|c} 16 & -12 & 36 \\ -12 & 20 & -24 \end{array}\right] \sim \begin{bmatrix} 4 & -3 & 9 \\ -3 & 5 & -6 \end{bmatrix} \sim \begin{bmatrix} 4 & -3 & 9 \\ 0 & 2.75 & 0.75 \end{bmatrix}$$

$I_3 = 0.2727$

$I_1 = 2.4545$

$I_2 = 2.1818$

1.4-2

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix} \sim \begin{bmatrix} a & b \\ 0 & a-\frac{b^2}{a} \end{bmatrix} \sim \begin{bmatrix} a & b \\ 0 & \frac{a^2-b^2}{a} \end{bmatrix}$$ for $a \neq 0$ rank is 2

basis for row space $\left\{ [a \ \ b], \left[0 \ \ \frac{a^2-b^2}{a}\right] \right\}$

column

$$A^T = \begin{bmatrix} a & b \\ b & a \end{bmatrix}^T \sim \begin{bmatrix} a & b \\ 0 & \frac{a^2-b^2}{a} \end{bmatrix}^T \sim \begin{bmatrix} a & 0 \\ b & \frac{a^2-b^2}{a} \end{bmatrix}$$ rank is 2

basis for column space $\left\{ \begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{a^2-b^2}{a} \end{bmatrix} \right\}$

7.4-9

$$\begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 1 & 1-\frac{1}{9} & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 9 & 8 & 9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ rank is 3

basis for row space $\{[9 \; 0 \; 1 \; 0], [0 \; 9 \; 8 \; 9], [0 \; 0 \; 1 \; 0]\}$

column

Symmetric!

$$A^T = \begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}^T \sim \begin{bmatrix} 9 & 0 & 1 & 0 \\ 0 & 9 & 8 & 9 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T$$ rank is 3

basis for column space $\left\{ \begin{bmatrix} 9 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 9 \\ 8 \\ 9 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

$\boxed{1.4\text{-}12}$ Show rank $\bar{B}^T\bar{A}^T$ = rank $\bar{A}\bar{B}$

from Theorem 3 (p.284) " $\bar{A}$ and its transpose $\bar{A}^T$ have the same rank"

and use of the property $(\bar{A}\bar{B})^T = \bar{B}^T\bar{A}^T$

rank $\bar{A}\bar{B}$ = rank $(\bar{A}\bar{B})^T$ (Theorem 3)

rank $(\bar{A}\bar{B})^T$ = rank $\bar{B}^T\bar{A}^T$

thus rank $\bar{B}^T\bar{A}^T$ = rank $\bar{A}\bar{B}$

$\boxed{1.4-14}$ Show: If $\bar{A}$ is not square, either the row vectors or the column vectors of A are linearly dependent.

From p. 284, Theorem 3 - "The rank r of a matrix $\bar{A}$ equals the maximum number of linearly independent column vectors of $\bar{A}$. Hence $\bar{A}$ and its transpose $\bar{A}^T$ have the same rank."

If $\bar{A}$ is $m \times n$ with $m > n$, then the rank $A \leq n < m$ thus the m row vectors are linearly dependent.

Similarly, if $\bar{A}$ is $m \times n$ with $m < n$, then the rank $A \leq m < n$ thus the n column vectors are linearly dependent.

We conclude that if $\bar{A}$ is not square, either the row vectors or the column vectors of $\bar{A}$ are linearly dependent.

$\boxed{1.4\text{-}15}$ ✓ Show: If the row vectors of a square matrix are LI, so are the columns, and conversely.

Using Theorem 3 "$\bar{A}$ and its transpose $\bar{A}^T$ have the same rank"

$\bar{A}_{n\times n}$ is LI in row vectors and therefore has rank $\bar{A} = n$

by Theorem 3, $\bar{A}^T_{n\times n}$ has rank $\bar{A} = n$ in terms of row vectors

The row vectors of $\bar{A}^T_{n\times n}$ are the column vectors of $\bar{A}_{n\times n}$.

Thus both the row vectors and column vectors of a square matrix are LI if either one is LI.

$\boxed{7.4-17}$ ✓

$$\begin{bmatrix} 3 & 4 & 0 & 2 \\ 2 & -1 & 3 & 7 \\ 1 & 16 & -12 & -22 \end{bmatrix} \sim \begin{bmatrix} 3 & 4 & 0 & 2 \\ 0 & -11/3 & 3 & 17/3 \\ 0 & \frac{44}{3} & -12 & \frac{-68}{3} \end{bmatrix} \sim \begin{bmatrix} 3 & 4 & 0 & 2 \\ 0 & -11/3 & 3 & 17/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Row 3 can be represented by a weighted combination of rows 2 and 3. The columns are $\boxed{\text{not linearly independent.}}$

1.4-52

Is the given set of vectors a vector space?

From Greenberg "Adv. Engr. Math, 2nd Edition" p. 452

"We call a nonempty set S of objects a vector space if the following requirements are met:

- Vector addition is defined between any two vectors in S in such a way that if $\bar{u}$ and $\bar{v}$ are in S, then $\bar{u}+\bar{v}$ is too (i.e. S is closed under addition)

- S contains a unique zero vector $\bar{0}$ such that $\bar{u}+\bar{0}=\bar{u}$ for each $\bar{u}$ in S

- For each $\bar{u}$ in S there is a unique vector $-\bar{u}$ in S such that $\bar{u}+(-\bar{u})=\bar{0}$

- Scalar multiplication is defined such that if $\bar{u}$ is any vector in S and α is any scalar, then the scalar multiple $\alpha\bar{u}$ is in S (i.e. S is closed under scalar multiplication)

All vectors in $\mathbb{R}^3$ with $3v_1 - 2v_2 + v_3 = 0$ $\quad 4v_1 + 5v_2 = 0$

~~vector~~ set S is closed under addition → yes

set S is closed under scalar multiplication → yes

The vectors are a vector space.

$$\begin{bmatrix} 3 & -2 & 1 \\ 4 & 5 & 0 \end{bmatrix} \sim \begin{bmatrix} 3 & -2 & 1 \\ 0 & \frac{23}{3} & -4/3 \end{bmatrix}$$ dimension 2

$v_1 = \frac{31}{23}v_3$

$v_2 = \frac{4}{23}v_3$

v_3 = arbitrary

1.4-34 Is the given set of vectors a vector space?

All vectors in $\mathbb{R}^n$ with $|v_j| = 1$ for $j = 1 \dots n$

· Vector addition is closed → not satisfied

ex. $\bar{a} = [1 \ 1 \ 1 \ 1 \ 1 \ \dots]$

$\bar{a} + \bar{a} = [2 \ 2 \ 2 \ 2 \ \dots]$ which does not meet $|v_j| = 1$

· To further show it is not a vector space, consider the zero vector requirement. $\bar{0}$ cannot reside within $|v_j| = 1$

$\boxed{\text{Not a vector space}}$