#12. Given :-

$$C = \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix}, \quad E = \begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix}$$

To find :-

(i) $(C+D)+E$ (ii) $(D+E)+C$ (iii) $0(C-E)+4D$

(iv) $A-0C$

Solution :-

(i) $$(C+D)+E = \underbrace{\begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix}}_{(C+D)} + \underbrace{\begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix}}_{E}$$

$$= \begin{bmatrix} 1 & 3 \\ 3 & 4 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix}$$

$$(C+D)+E = \begin{bmatrix} 1 & 5 \\ 6 & 8 \\ 6 & -2 \end{bmatrix}$$ —— Ans.

(ii) $(D+E)+C = ?$

$$(D+E) = \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 3 \\ 8 & 4 \\ 5 & -2 \end{bmatrix}$$

$$\therefore (D+E)+C = \begin{bmatrix} -4 & 3 \\ 8 & 4 \\ 5 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 5 \\ 6 & 8 \\ 6 & -2 \end{bmatrix}$$ — Ans.

(iii) $0(C-E)+4D = ?$

$$(C-E) = \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 \\ -5 & 0 \\ -2 & 1 \end{bmatrix}$$

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(iii) $\therefore 0(C-E) = 0\begin{bmatrix} 5 & 0 \\ -5 & 0 \\ -2 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ ... if $A = [a_{jk}]_{m\times n}$ then $cA = [c\,a_{jk}]_{m\times n}$

$\therefore 0(C-E) + 4D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} + 4\begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} -16 & 4 \\ 20 & 0 \\ 8 & -4 \end{bmatrix}$ — Ans.

(iv) $A - 0C = ?$

$A - 0C = \begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix} - 0\begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}$

$= \underset{3\times \underline{3}}{\begin{bmatrix} 0 & 2 & 4 \\ 6 & 5 & 5 \\ 1 & 0 & -3 \end{bmatrix}} - \underset{3\times \underline{2}}{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}}$

Two matrices can be added or subtracted if & only if their dimensions match.
~~Thus,~~ However, here, first matrix i.e. A has 3×3 dimensions
& C has 3×2 dimensions.

Thus subtraction cannot be performed.

#13. Given :-

$$C = \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} -4 & 1 \\ 5 & 0 \\ 2 & -1 \end{bmatrix}, E = \begin{bmatrix} 0 & 2 \\ 3 & 4 \\ 3 & -1 \end{bmatrix} \& \ u = \begin{bmatrix} 1.5 \\ 0 \\ -3.0 \end{bmatrix}$$

To find :-

(i) $(2.7)C$ (ii) $2(7C)$ (iii) $-D + 0E$ (iv) $E - D + C + u$

Solution :-

(i) $$(2.7)C = (2.7)\begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}$$

$$= 14 \begin{bmatrix} 5 & 2 \\ -2 & 4 \\ 1 & 0 \end{bmatrix}$$

As no. of columns of first matrix C are not equal to no. of rows of second matrix i.e. a^T, matrix multiplication cannot be performed.

#29. Given :-

Profit for Sofas(S) :- \$35

for Chairs(C) :- \$62

for Tables(T) :- \$30

Sales matrix is

$$A = \begin{array}{c} \\ \\ \end{array}\begin{array}{c} \begin{array}{ccc} S & C & T \end{array} \\ \begin{bmatrix} 400 & 60 & 240 \\ 100 & 120 & 500 \end{bmatrix} \end{array}\begin{array}{l} \\ F_1 \\ F_2 \end{array}$$

Solution :-

$$\text{Profit vector 'p'} = \begin{bmatrix} 35 \\ 62 \\ 30 \end{bmatrix} \quad \text{—— Ans. (i)}$$

Thus total profit vector would be derived from the following relation :-

Total profit = Total sales × profit.

Thus, total profit vector $V = A\,p$.

Thus,

$$V = \begin{bmatrix} 400 & 60 & 240 \\ 100 & 120 & 500 \end{bmatrix} \begin{bmatrix} 35 \\ 62 \\ 30 \end{bmatrix}$$

$$= \begin{bmatrix} 400 \times 35 + 60 \times 62 + 240 \times 30 \\ 100 \times 35 + 120 \times 62 + 500 \times 30 \end{bmatrix}$$

$$V = \begin{bmatrix} 24{,}920 \\ 25{,}940 \end{bmatrix}$$ — Ans.

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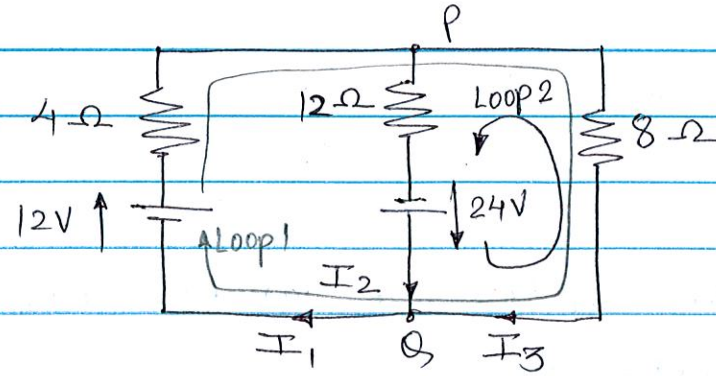
Please Turn Over

#18.



Solution :-

Considering Loop 1 & applying Kirchoff's Law,

$$12 - 4I_1 - 8I_3 = 0$$

$$\therefore \quad I_1 + 2I_3 = 3 \qquad \text{— (i)}$$

Similarly applying Kirchoff's Law for Loop 2,

$$24 - (-I_3 \times 8) - 12I_2 = 0.$$

$$\therefore \quad 24 + 8I_3 - 12I_2 = 0.$$

$$\therefore \quad 3I_2 - 2I_3 = 6 \quad \text{— (ii)}$$

And at node Q, we have

$$I_1 = I_2 + I_3$$

$$\text{or} \quad I_1 - I_2 - I_3 = 0 \qquad \text{— (iii)}$$

Putting above 3 equations in matrix form, we have,

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 3 & -2 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 0 \end{bmatrix}$$

Thus augmented matrix becomes,

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 3 & -2 & 6 \\ 1 & -1 & -1 & 0 \end{array}\right]$$

By $R_3 - R_1$,

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 3 & -2 & 6 \\ 0 & -1 & -3 & -3 \end{array}\right]$$

By $3R_3 + R_2$,

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 3 & -2 & 6 \\ 0 & 0 & -11 & -3 \end{array}\right]$$

This is row-echelon form / upper triangular matrix.

P.T.O. →

Thus R_3 gives,

$$-11 I_3 = -3$$

$$\therefore \quad I_3 = \frac{3}{11} \text{ Amp.}$$

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Back substituting in R_2,

$$3 I_2 - 2 \times \frac{3}{11} = 6.$$

$$\therefore \quad 3 I_2 = 6 + \frac{6}{11}$$

$$\therefore \quad I_2 = \frac{24}{11} \text{ Amp.}$$

& by back substituting these in R_1,

$$I_1 + 2 \times \frac{3}{11} = 3$$

$$\therefore \quad I_1 = \frac{27}{11} \text{ Amp.}$$

Thus

$$\boxed{I_1 = \frac{27}{11} \text{ Amp}, \quad I_2 = \frac{24}{11} \text{ Amp} \ \& \ I_3 = \frac{3}{11} \text{ Amp.}}$$

Thus the given set of vector is not linearly independent.

#32. Given :- For R^3

$$3v_1 - 2v_2 + v_3 = 0.$$
$$4v_1 + 5v_2 = 0.$$

To prove whether the given set is vector space or not, we need to find any relation between v_1, v_2 & v_3 if any.

For this purpose, above system of equation is expressed in matrix form.

$$\begin{bmatrix} 3 & -2 & 1 \\ 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ Coefficient matrix :-

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 4 & 5 & 0 \end{bmatrix}$$

By ~~$3R_2 - 4$~~ $3R_2 - 4R_1$,

$$= \begin{bmatrix} 3 & -2 & 1 \\ 0 & 23 & -4 \end{bmatrix}$$

Thus $\quad 23V_2 - 4V_3 = 0$

or $\quad V_2 = \frac{4}{23} V_3.$

& $\quad 3V_1 - 2\left(\frac{4}{23} V_3\right) + V_3 = 0.$

$\therefore \quad 3V_1 - \frac{8}{23} V_3 + V_3 = 0$

$\therefore \quad V_1 = \frac{-5}{23} V_3$

Thus vector space can be defined as

$$\vec{V} = \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} -5/23 \\ 4/23 \\ 1 \end{pmatrix} V_3.$$

Thus given set of vectors is a vector space of dimension 3.

$\therefore \quad \vec{V} \in R^3.$

$\dim N_A$ = no. of columns of A − rank(A)

$= 3 - 2$

$= 1$

$\therefore \quad N_A = \text{span}\left\{\left[\frac{-5}{23} \quad \frac{4}{23} \quad 1\right]\right\}$

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#34. To ~~pr~~ find :-

whether all vectors in R^n with $|v_j| = 1$ for $j = 1, \dots, n$

is vector space or not.

Solution :-

$$\left\{ \vec{v} \in R^n \;\middle|\; |v_i| = 1 \right\}$$

Where $\vec{v} = (\pm 1, \dots \pm 1)$.

As all the elements in given set of vectors are ± 1, it does not contain $\vec{0}$ in it.

Any set of vectors must complete this condition of existence of $\vec{0}$ in it ~~as~~ to be a vector space.

Thus given set of vectors is not a vector space.

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