Moses Navarro
Math 527

Homework # 6 -

Problem set 6.1, pg. 210

Laplace Transforms

(1) $3t + 12 = f(t)$

$$\mathcal{L}(f) = \int_0^\infty e^{-st}[3t+12]\,dt = 3\int_0^\infty e^{-st}t + 12\int e^{-st}\,dt$$

For 1st integral, set $f' = e^{-st}$ & $g = t \Rightarrow f = -\frac{e^{-st}}{s}$ $g' = 1 \Rightarrow \int f'g\,dt = fg - \int fg'\,dt$

$$\mathcal{L}(f) = 3\left[\left[-\frac{e^{-st}}{s}t\right]_0^\infty + \int_0^\infty \frac{e^{-st}}{s}\,dt\right] + 12\left[-\frac{e^{-st}}{s}\right]_0^\infty dt$$

0

$$= 3\left[0 + \left[-\frac{e^{-st}}{s^2}\right]_0^\infty\right] + \frac{12}{s}$$

$$\boxed{\mathcal{L}(f) = \frac{3}{s^2} + \frac{12}{s}}$$

Moses Navarro
Math 527

Homework # 6 Continued

(2) $f(t) = (a-bt)^2 = (a-bt)(a-bt) = a^2 - 2abt + b^2t^2$

$$\mathcal{L}[f] = \int_0^\infty e^{-st}\left[a^2 - 2abt + b^2t^2\right]dt$$

$$= \int_0^\infty e^{-st}a^2\,dt - \int_0^\infty e^{-st}\,2abt\,dt + \int_0^\infty e^{-st}b^2t^2\,dt$$

$$= a^2\underbrace{\int_0^\infty e^{-st}\,dt}_{1st} - 2ab\underbrace{\int_0^\infty e^{-st}t\,dt}_{2nd} + b^2\underbrace{\int_0^\infty e^{-st}t^2\,dt}_{3rd}$$

Find solution to 3rd Integral

$f' = e^{-st}$ + $g = t^2$ $\Rightarrow$ $f = -\dfrac{e^{-st}}{s}$ $\quad g' = 2t$

$$\int_0^\infty e^{-st}t^2\,dt = \left[\cancelto{0}{-\frac{e^{-st}}{s}t^2}\right]_0^\infty - \int_0^\infty\left(-\frac{e^{-st}}{s}\right)2t\,dt = 2\int_0^\infty \frac{e^{-st}}{s}t\,dt$$

$f' = \dfrac{e^{-st}}{s}$ $\quad g = t$ $\quad f = -\dfrac{e^{-st}}{s^2}$ $\quad g' = 1$

$$2\int_0^\infty \frac{e^{-st}}{s}t\,dt = 2\left[\left[\cancelto{0}{-\frac{e^{-st}}{s^2}t}\right]_0^\infty - \int_0^\infty\left(-\frac{e^{-st}}{s^2}\right)dt\right] = -2\left[\frac{e^{-st}}{s^3}\right]_0^\infty = \frac{2}{s^3}$$

$$\mathcal{L}(f) = a^2\left[-\frac{e^{-st}}{s}\right]_0^\infty - 2ab\left[-\frac{e^{-st}}{s^2}\right]_0^\infty - 2b^2\left[\frac{e^{-st}}{s^3}\right]_0^\infty$$

$$\boxed{\mathcal{L}(f) = \frac{a^2}{s} - \frac{2ab}{s^2} + \frac{2b^2}{s^3}}$$

Moses Novallo
Math 527

Homework # 6 Continued

5. $f(t) = e^{2t}\sinh t = e^{2t}\left[\frac{1}{2}\left(-e^{-t} + e^{t}\right)\right]$

$$f(t) = \frac{1}{2}\left[-e^{t} + e^{3t}\right]$$

$$\mathcal{L}(s) = \int_0^\infty e^{-st} f(t)\,dt = \int_0^\infty e^{-st}\left[\frac{1}{2}\left(-e^{t} + e^{3t}\right)\right] dt$$

$$= \frac{1}{2}\int_0^\infty \left[-e^{-st+t} + e^{-st+3t}\right] dt$$

$$= -\frac{1}{2}\int_0^\infty e^{(1-s)t}\,dt + \frac{1}{2}\int_0^\infty e^{(3-s)t}\,dt$$

$$= -\frac{1}{2}\left[\frac{e^{(1-s)t}}{(1-s)}\right]_0^\infty + \frac{1}{2}\left[\frac{e^{(3-s)t}}{(3-s)}\right]_0^\infty$$

↓ for $s>1$ ↓ for $s>3$

$$= \left(\frac{1}{2}\right)\left[\frac{1}{(1-s)}\right] - \frac{1}{2}\left[\frac{1}{(3-s)}\right] = \frac{1}{2}\left[\frac{(3-s)-(1-s)}{(1-s)(3-s)}\right]$$

$$= \frac{1}{2}\left[\frac{3-\cancel{s}-1+\cancel{s}}{3-4s+s^2}\right] = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{s^2-4s+3}\right] = \frac{1}{s^2-4s+3}$$

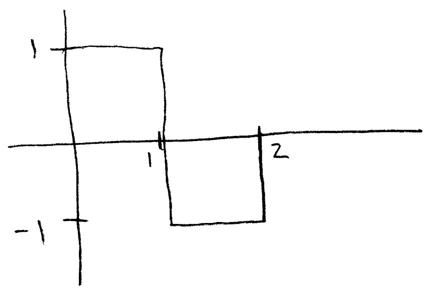
$$\boxed{\mathcal{L}(s) = \frac{1}{s^2-4s+3} \quad \text{or} \quad \frac{1}{(s-2)^2-1}}$$

Moses Navarro
Math 527

4/20

Homework # 6 Continued

13.) $f(t)$ defined as follows



$f(t) = 1 \quad , \quad 0 < t < 1$

$f(t) = -1 \quad , \quad 1 < t < 2$

$f(t) = 0 \quad , \quad t > 2$

$$\mathcal{L}(s) = \int_0^1 e^{-st}(1)\,dt + \int_1^2 e^{-st}(-1)\,dt + \cancel{\int_2^{\infty} e^{-st}(0)\,dt} \to 0$$

$$= \left[-\frac{e^{-st}}{s}\right]_0^1 - \left[-\frac{e^{-st}}{s}\right]_1^2$$

$$= \left(-\frac{e^{-s}}{s} + \frac{1}{s}\right) - \left(-\frac{e^{-2s}}{s} + \frac{e^{-s}}{s}\right)$$

$$= -\frac{e^{-s}}{s} + \frac{1}{s} + \frac{e^{-2s}}{s} - \frac{e^{-s}}{s} = \boxed{\frac{e^{-2s}}{s} - \frac{2e^{-s}}{s} + \frac{1}{s} = \mathcal{L}(s)}$$

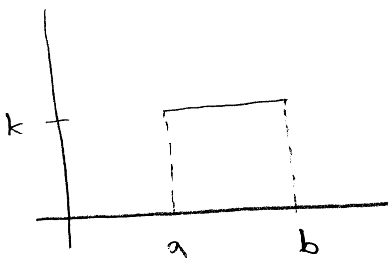
or

$$\boxed{\mathcal{L}(s) = \frac{e^{-2s} - 2e^{-s} + 1}{s} = \frac{(e^{-s}-1)^2}{s}}$$

Moses Navarro
Math 527

Homework # 6 Continued

(14.) $f(t)$ defined as follows



$f(t) = 0 \,, \quad 0 < t < a \quad \& \quad t > b$

$f(t) = k \,, \quad a < t < b$

$$\mathcal{L}(f) = \int_a^b e^{-st} k \, dt = k \int_a^b e^{-st} \, dt = k \left[-\frac{e^{-st}}{s} \right]_a^b$$

$$\boxed{\mathcal{L}(f) = k \left[-\frac{e^{-bs}}{s} + \frac{e^{-as}}{s} \right] = k \left(\frac{e^{-as} - e^{-bs}}{s} \right)}$$

Moses Navarro
Math 527
6/20

Homework # 6 Continued

Some Theory

(23.) Change of scale, If $\mathcal{L}(f(t)) = F(s)$ and c is any positive constant, show that $\mathcal{L}(f(ct)) = F(s/c)/c$

when $t=0$, $p=0$

Set $p = ct$, then $dp = c\,dt \Rightarrow dt = \frac{dp}{c}$

$$(1) = \mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\,dt$$

Change of variable

$$= \mathcal{L}\left(f\left(\tfrac{p}{c}\right)\right) = \int_0^\infty e^{-\frac{sp}{c}} \frac{dp}{c} = \frac{1}{c}\int_0^\infty e^{-s\left(\frac{p}{c}\right)}\, dp = \frac{F(s/c)}{c}$$

$$\boxed{\mathcal{L}(f(p/s)) = \frac{F(s/c)}{c}}$$

Moses Navarro
Math 527
7/20

Homework # 6 Continued

Inverse Laplace transform

Given $F(s) = \mathcal{L}(s)$, find $f(t)$, a, b, L, n are constants.

(30) $$F(s) = \frac{4s+32}{s^2-16} = \frac{4s}{s^2-(4)^2} + \frac{8(4)}{s^2-(4)^2}$$

$$f(t) = \mathcal{L}^{-1}\left[F(s)\right] = 4\,\mathcal{L}^{-1}\left[\frac{s}{s^2-4^2}\right] + 8\,\mathcal{L}^{-1}\left[\frac{4}{s^2-4^2}\right]$$

$$\boxed{f(t) = 4\cosh(4t) + 8\sinh(4t)}$$

(32) $$\frac{1}{(s+a)(s+b)} = \frac{A}{(s+a)} + \frac{B}{s+b} = \frac{A(s+b) + B(s+a)}{(s+a)(s+b)}$$

$$1 = As + Ab + Bs + Ba = (A+B)s + Ab + Ba$$

$$0 = (A+B)s \Rightarrow A = -B \quad \& \quad 1 = Ab + Ba = Ab - Aa = A(b-a)$$
$$\Rightarrow A = \frac{1}{b-a}$$

$$\frac{1}{(s+a)(s+b)} = \frac{1}{(b-a)(s+a)} - \frac{1}{(b-a)(s+b)} = \frac{1}{b-a}\left[\frac{1}{s+a} - \frac{1}{s+b}\right] = F(s)$$

$$f(t) = \mathcal{L}^{-1}\left[F(s)\right] = \frac{1}{b-a}\left[\mathcal{L}^{-1}\left[\frac{1}{s+a}\right] - \mathcal{L}^{-1}\left[\frac{1}{s+b}\right]\right]$$

$$\boxed{f(t) = \frac{1}{b-a}\left[e^{-at} - e^{-bt}\right]}$$

Moses Navarro

Math 527

8/20

Homework # 6 Continued

Problem Set 6.2, pg 216

Initial Value Problems

(4.) $y'' + 9y = 10e^{-t}$, $y(0) = 0$, $y'(0) = 0$

$\mathcal{L}(y'') = s^2 Y(s) - s y(0) - y'(0) = s^2 Y(s)$

$\mathcal{L}(y) = Y(s)$

$\mathcal{L}(e^{-t}) = \dfrac{1}{s+1}$

$$s^2 Y(s) + 9Y(s) = \frac{10}{s+1}$$

$$(s^2+9)Y(s) = \frac{10}{s+1} \Rightarrow Y(s) = \frac{10}{(s^2+(3)^2)(s+1)}$$

$$\frac{10}{(s^2+9)(s+1)} = \frac{As+B}{s^2+9} + \frac{C}{s+1} = \frac{(As+B)(s+1)\ C(s^2+9)}{(s^2+9)(s+1)}$$

$10 = As^2 + As + Bs + B + Cs^2 + 9C$

$0 = (A+C)s^2 \Rightarrow A = -C \Rightarrow C = -A$

$0 = (A+B)s \qquad A = -B \Rightarrow B = -A$

$10 = B + 9C \Rightarrow 10 = -A - 9A = -10A \Rightarrow A = -1$

$C = 1$, $B = 1$

Moses Navarro
Math 527

Homework #6 Continued

(4.) Continued

$$Y(s) = \frac{10}{(s^2+9)(s+1)} = \frac{-s+1}{s^2+9} + \frac{1}{s+1} = \frac{1}{s^2+9} - \frac{s}{s^2+9} + \frac{1}{s+1}$$

$$Y(s) = \frac{1}{3}\frac{3}{s^2+3^2} - \frac{s}{s^2+3^2} + \frac{1}{s+1}$$

$$y(t) = \mathcal{L}^{-1}\left[Y(s)\right] = \boxed{\frac{1}{3}\sin(3t) - \cos(3t) + e^{-t} = y(t)}$$

Moses Navarro
Math 527
10/20

Homework # 6 Continued

(5.) $y'' - \frac{1}{4}y = 0$, $y(0) = 12$, $y'(0) = 0$

$$\mathcal{L}(y'') = s^2 Y(s) - s\,y(0) - y'(0) = s^2 Y - 12s$$

$$\mathcal{L}\left(-\frac{1}{4}y\right) = -\frac{1}{4} Y$$

$$s^2 Y - 12s - \frac{1}{4}Y = 0 \Rightarrow \left(s^2 - \frac{1}{4}\right)Y = 12s$$

$$Y(s) = \frac{12s}{\left(s^2 - \left(\frac{1}{2}\right)^2\right)} = 12\left[\frac{s}{\left(s^2 - \left(\frac{1}{2}\right)^2\right)}\right]$$

$$y(t) = \mathcal{L}^{-1}\left[Y(s)\right] = 12\cosh\left(\frac{t}{2}\right) = 12\left[\frac{1}{2}\left(e^{-t/2} + e^{t/2}\right)\right]$$

$$\boxed{y(t) = 12\cosh\left(\frac{t}{2}\right) = 6\left[e^{-t/2} + e^{t/2}\right]}$$

Moses Navarro
Math 527
11/20

Homework # 6 Continued

Obtaining Transforms by Differentiation

Find $\mathcal{L}(f)$

17. $f(t) = te^{-at}$ $\quad f(0) = 0$

$$f'(t) = e^{-at} - a\underbrace{te^{-at}}_{f} \quad f'(0) = 1 - 0 = 1$$

$$\mathcal{L}(f') = \frac{1}{s+a} - a\mathcal{L}(f) = s\mathcal{L}(f) - \cancel{f(0)} = s\mathcal{L}(f)$$

$$= \mathcal{L}(f)(s+a) = \frac{1}{s+a} \Rightarrow \boxed{\mathcal{L}(f) = \frac{1}{(s+a)^2}}$$

19. $f(t) = \sin^2 \omega t$ $\quad f(0) = 0$

$$f' = 2\sin(\omega t)\cos(\omega t)\,\omega = 2\omega \sin(\omega t)\cos(\omega t) = \omega\left[2\sin(\omega t)\cos(\omega t)\right]$$

use trig identity

$$\Longrightarrow f' = \omega(\sin(2\omega t))$$

$$\mathcal{L}(f') = \omega\left(\frac{2\omega}{s^2 + (2\omega)^2}\right) = s\mathcal{L}(f) - \cancel{f(0)}^{\,0}$$

$$\boxed{\mathcal{L}(f) = \left(\frac{1}{s}\right)\left(\frac{2\omega^2}{s^2 + 4\omega^2}\right) = \frac{2\omega^2}{s^3 + 4\omega^2 s}}$$

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Math 527

Homework # 6 Continued

Inverse Transforms by Integration

(26.) $\mathcal{L}[F] = \dfrac{1}{s^4 - s^2} = \dfrac{1}{s^2(s^2-1)}$

$\mathcal{L}^{-1}\left[\dfrac{1}{(s^2-1)}\right] = \sinh t$, $\mathcal{L}^{-1}\left[\dfrac{1}{s(s^2-1)}\right] = \displaystyle\int_0^t \sinh t \, dt = \cosh(t) - \cosh(0)^{\swarrow 1}$

$= \cosh(t) - 1$

$$\mathcal{L}^{-1}\left[\frac{1}{s^2(s^2-1)}\right] = \int_0^t [\cosh(\tau) - 1]\, dt = \Big[\sinh(\tau) - \tau\Big]_0^t = \sinh(t) - t$$

$$\boxed{\mathcal{L}^{-1}\left[\frac{1}{s^4 - s^2}\right] = \sinh(t) - t = \frac{1}{2}(-e^{-t} + e^t) - t}$$

Moses Navallo
Math 527

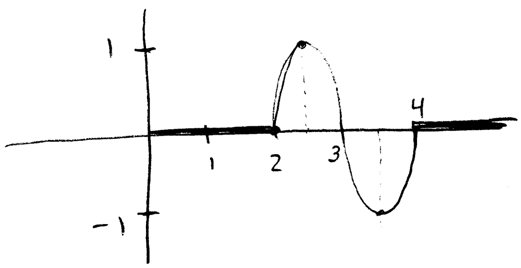
Homework # 6 Continued

Problem Set 6.3, pg. 223

Second shifting theorem, unit step function

Sketch or graph the given function, which is assumed to be zero outside the given interval. Represent it, using unit step functions. Find its transform.

(6) $f(t) = \sin \pi t \quad , \quad 2 < t < 4$



$$\boxed{f(t) = \sin \pi t \left[u(t-2) - u(t-4) \right]}$$

$$\mathcal{L}(f) = \mathcal{L}\left[\sin \pi t \, (u(t-2) - u(t-4)) \right]$$

$\mathcal{L}\left[\sin(\pi t)\, u(t-2) \right]$ Find $f(t-a)$

$$\sin(\pi t) = \sin(\pi(t-2+2)) = \sin(\pi(t-2) + 2\pi)$$
$$= \sin(\pi(t-2))\cos(2\pi) + \cos(\pi(t-2))\cancel{\sin(2\pi)}$$

$$\sin(\pi t) = \sin(\pi(t-2))$$
$$\mathcal{L}\left[\sin(\pi(t-2))\, u(t-2) \right] = e^{-2s}\left(\frac{\pi}{s^2+\pi^2} \right)$$

$$-\mathcal{L}\left[\sin(\pi t)\, u(t-4) \right] = -\mathcal{L}\left[\sin(\pi(t-4))\, u(t-4) \right] = -e^{-4s}\left(\frac{\pi}{s^2+\pi^2} \right)$$

$$\boxed{\mathcal{L}(f) = e^{-2s}\left[\frac{\pi}{s^2+\pi^2} \right] - e^{-4s}\left[\frac{\pi}{s^2+\pi^2} \right] = \left[\frac{\pi}{s^2+\pi^2} \right]\left(e^{-2s} - e^{-4s} \right)}$$

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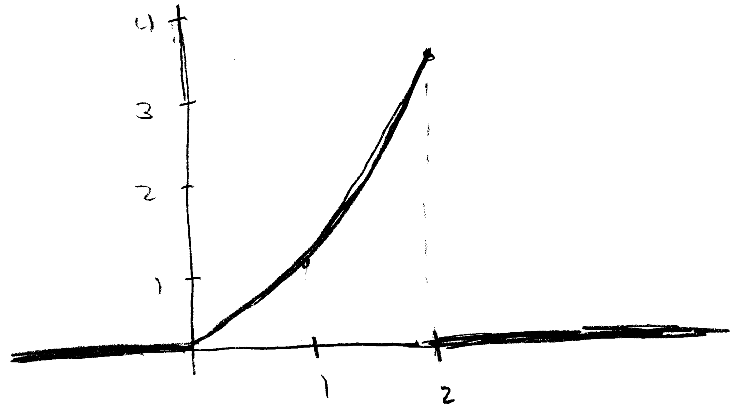
Math 527
14/20

Homework # 6 Continued

(10.) $\sinh(t)$, $0 < t < 2$



$$f(t) = \sinh(t)[1 - u(t-2)] = \sinh(t) - \sinh(t)u(t-2)$$

$$\sinh(t-2+2) = \sinh(t-2)\cosh(2) + \cosh(t-2)\sinh(2)$$

$$f(t) = \sinh(t) - [\cosh(2)\sinh(t-2)u(t-2) + \sinh(2)\cosh(t-2)u(t-2)]$$

$$f(t) = \sinh(t) - \cosh(2)\sinh(t-2)u(t-2) - \sinh(2)\cosh(t-2)u(t-2)$$

$$\Rightarrow \mathcal{L}[\sinh(t)] = \frac{1}{s^2-1}$$

$$\Rightarrow -\cosh(2)\mathcal{L}[\sin(t-2)u(t-2)] = -\cosh(2)\left[\frac{1}{s^2-1}\right]e^{-2s}$$

$$\Rightarrow -\sinh(2)\mathcal{L}[\cosh(t-2)u(t-2)] = -\sinh(2)\left[\frac{s}{s^2-1}\right]e^{-2s}$$

$$\mathcal{L}[s] = \frac{1}{s^2-1} - \frac{\cosh(2)}{s^2-1}e^{-2s} - \frac{\sinh(2)s}{s^2-1}e^{-2s}$$

$$= \frac{1}{s^2-1}\left[1 - \cosh(2)e^{-2s} - \sinh(2)s\,e^{-2s}\right]$$

Moses Navarro
Math 527

Homework # 6 Continued

(10.) Continued

$$\cosh(2) = \frac{1}{2}\left[e^{-2} + e^{2}\right]$$

$$\cosh(2)e^{-2s} = \frac{1}{2}\left[e^{-2s-2} + e^{-2s+2}\right] = \frac{1}{2}\left[e^{-2(s+1)} + e^{-2(s-1)}\right]$$

$$\sinh(2) = \frac{1}{2}\left[-e^{-2} + e^{2}\right]$$

$$\sinh(2)e^{-2s}s = \frac{s}{2}\left[-e^{-2(s+1)} + e^{-2(s-1)}\right]$$

$$y[s] = \frac{1}{s^2-1}\left[1 - \frac{e^{-2(s+1)}}{2} - \frac{e^{-2(s-1)}}{2} + \frac{se^{-2(s+1)}}{2} - \frac{se^{-2(s-1)}}{2}\right]$$

$$\boxed{y[s] = \frac{1}{s^2-1}\left[1 - \frac{e^{-2(s+1)}}{2}(1-s) - \frac{e^{-2(s-1)}}{2}(1+s)\right]}$$

Moses Navarro
Math 527

Homework #16 Continued

Inverse Transforms by the 2nd shifting Theorem

Find and sketch or graph $f(t)$.

(13.) $\mathcal{L}(f) = \dfrac{6(1-e^{-\pi s})}{s^2+9} = \dfrac{6}{s^2+(3)^2} - \left[\dfrac{6}{s^2+(3)^2}\right] e^{-\pi s}$

$f = \mathcal{L}^{-1}(F) = 2\,\mathcal{L}^{-1}\left[\dfrac{3}{s^2+(3)^2}\right] - 2\,\mathcal{L}^{-1}\left[\dfrac{3}{s^2+(3)^2}\, e^{-\pi s}\right]$

$= 2\sin(3t) - 2\sin(3(t-\pi))\, u(t-\pi)$

$\sin(3(t-\pi)) = \sin(3t-3\pi) = \sin(3t)\cos(3\pi) - \cos(3t)\cancel{\sin(3\pi)}^{\,0}$

$\sin(3(t-\pi)) = -\sin(3t)$

$f = 2\sin(3t) + 2\sin(3t)\,u(t-\pi) = \boxed{2[1+u(t-\pi)]\sin(3t)}$

Moses Navarro

Math 527

17/20

Homework #6 continued

(16.) $\mathcal{L}(f) = \dfrac{2(e^{-s} - e^{-3s})}{s^2 - 4} = \dfrac{2e^{-s} - 2e^{-3s}}{s^2 - (2)^2} = \dfrac{2e^{-s}}{s^2 - (2)^2} - \dfrac{2e^{-3s}}{s^2 - (2)^2}$

$f = \mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1}\left[\dfrac{2}{s^2 - (2)^2} e^{-s}\right] - \mathcal{L}^{-1}\left[\dfrac{2}{s^2 - (2)^2} e^{-3s}\right]$

$$\boxed{f = \sinh(2(t-1))u(t-1) - \sinh(2(t-3))u(t-3)}$$

Moses Navarro
Math 527

Homework #6 Continued

IVPs, some with Discontinous

(25.) $y'' + y = t$ if $0 < t < 1$ and 0 if $t > 1$

$y(0) = 0$, $y'(0) = 0$

$\mathcal{L}[y'' + y] = s^2 Y - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{0} + Y = s^2 Y + Y = Y(s^2+1)$

$R.H.S = t\,[1 - u(t-1)] = t - t\,u(t-1) = t - (t-1+1)\,u(t-1)$

$R.H.S = t - [(t-1)+1]\,u(t-1) = t - (t-1)u(t-1) - u(t-1)$

$\mathcal{L}[R.H.S] = \dfrac{1}{s^2} - \dfrac{e^{-s}}{s^2} - \dfrac{e^{-s}}{s}$

$Y(s^2+1) = \dfrac{1}{s^2} - \dfrac{e^{-s}}{s^2} - \dfrac{e^{-s}}{s}$

$Y = \dfrac{1}{s^2(s^2+1)} - \dfrac{e^{-s}}{s^2(s^2+1)} - \dfrac{e^{-s}}{s(s^2+1)}$

Use Partial Fractions

$\dfrac{1}{s^2(s^2+1)} = \dfrac{As+B}{s^2} + \dfrac{Cs+D}{s^2+1} = \dfrac{(As+B)(s^2+1) + (Cs+D)(s^2)}{s^2(s^2+1)}$

$1 = As^3 + Bs^2 + As + B + Cs^3 + Ds^2$

$1 = (A+C)s^3 + (B+D)s^2 + As + B$

$A + C = 0 \Rightarrow A = -C$ $\quad B + D = 0 \Rightarrow B = -D$ $\quad A = 0$ $\quad B = 1$

$\dfrac{1}{s^2(s^2+1)} = \dfrac{1}{s^2} + \dfrac{(-1)}{s^2+1} = \dfrac{1}{s^2} - \dfrac{1}{s^2+1}$

Moser Navarro
Math 527

Homework # 6 Continued

(25.) Continued

$$\boxed{\frac{e^{-s}}{s^2(s^2+1)} = \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s^2+1}}$$

$$\frac{e^{-s}}{s(s^2+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1} = \frac{A(s^2+1) + s(Bs+C)}{s(s^2+1)}$$

$$e^{-s} = As^2 + Bs^2 + Cs + A = (A+B)s^2 + Cs + A$$

$$C = 0 \qquad A = -B \qquad A = e^{-s}$$

$$\frac{e^{-s}}{s(s^2+1)} = \frac{e^{-s}}{s} + \frac{(-e^{-s})s}{s^2+1} = \boxed{\frac{e^{-s}}{s} - \frac{se^{-s}}{s^2+1}}$$

$$Y = \left[\frac{1}{s^2} - \frac{1}{s^2+1}\right] - \left[\frac{e^{-s}}{s^2} - \frac{e^{-s}}{s^2+1}\right] - \left[\frac{e^{-s}}{s} - \frac{se^{-s}}{s^2+1}\right]$$

$$Y = \frac{1}{s^2} - \frac{1}{s^2+1} - \frac{e^{-s}}{s^2} + \frac{e^{-s}}{s^2+1} - \frac{e^{-s}}{s} + \frac{se^{-s}}{s^2+1}$$

$$\mathcal{L}^{-1}[Y] = t - \sin(t) - (t-1)u(t-1) + \sin(t-1)u(t-1) - u(t-1) + \cos(t-1)u(t-1)$$

$$y(t) = t - \sin(t) - tu(t-1) + \cancel{u(t-1)} + \sin(t-1)u(t-1) - \cancel{u(t-1)} + \cos(t-1)u(t-1)$$

$$y(t) = t - \sin(t) - tu(t-1) + \sin(t-1)u(t-1) + \cos(t-1)u(t-1)$$

$$\boxed{y(t) = t - \sin(t) - u(t-1)\left[t - \sin(t-1) - \cos(t-1)\right]}$$

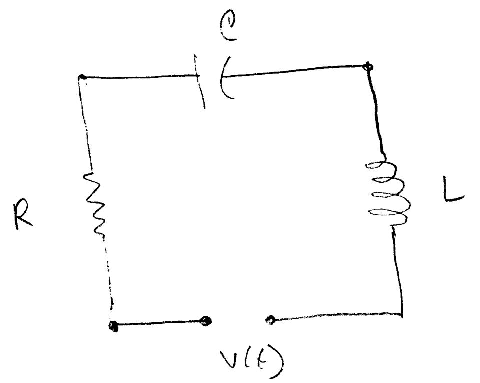
Moses Navello
Math 527
20/20

Homework # 6 continued

RLC- Circuit

Using the Laplace transform and showing the details, find the current $i(t)$.

(39.) $R = 2\,\Omega$, $L = 1\,H$, $C = 0.5\,F$, $v(t) = 1kV = 1{,}000V$ if $0 < t < 2$ and 0 if $t > 2$



$$L I' + R I + \frac{1}{C}\int I\,dt = E(t)$$

$$i' + 2i + \frac{1}{0.5}\int_0^t i\,d\tau = 1{,}000[1 - u(t-2)]$$

$$\mathcal{L}[L.H.S] = sI + 2I + 2\frac{I}{s} = I\left(s + 2 + \frac{2}{s}\right)$$

$$\mathcal{L}[R.H.S] = \frac{1000}{s} - 1000\left(\frac{e^{-2s}}{s}\right)$$

$$\mathcal{L}[L.H.S] = I\left[\frac{s^2+2s+2}{s}\right] = I\left[\frac{(s+1)^2+1}{s}\right]$$

$$I = \frac{s}{(s+1)^2+1}\left[\frac{1000}{s} - \frac{1000\,e^{-2s}}{s}\right] = \frac{1000}{(s+1)^2+1} - \frac{1000\,e^{-2s}}{(s+1)^2+1}$$

$$i = \mathcal{L}^{-1}[I] = 1000\,e^{-t}\sin(t) - 1000\,e^{-(t-2)}\sin(t-2)\,u(t-2)$$

$$i = 1000\,e^{-t}\sin(t) - 1000\,e^{-t+2}\sin(t-2)\,u(t-2)$$