so as $\mathcal{L}(\cos t) = \int_0^\infty e^{-st} \cos t \, dt = \int_0^\infty e^{-st} \, d\sin t = e^{-st} \sin t \Big|_0^\infty - s\int_0^\infty e^{-st} \, d\cos t$

$$= -s\left(e^{-st} \cos t \Big|_0^\infty + s\int_0^\infty e^{-st} \cos t \, dt \right)$$

$$= s - s^2 \int_0^\infty e^{-st} \cos t \, dt$$

$$\Rightarrow \quad \mathcal{L}(\cos t) = \frac{s}{s^2+1}$$

According to the above equation, $\mathcal{L}(\cos wt) = F\left(\frac{s}{w}\right) \cdot \frac{1}{w} = \frac{\frac{s}{w}}{\frac{s^2}{w^2}+1} \cdot \frac{1}{w} = \frac{s}{s^2+w^2}$

30. Answer:

$$F(s) = \frac{4s+32}{s^2-16} = \frac{2(s+4+s-4+16)}{(s+4)(s-4)} = \frac{6}{s-4} - \frac{2}{s+4}$$

hence $\mathcal{L}^{-1}(F(s)) = 6e^{4t} - 2e^{-4t}$

32. Answer:

$$F(s) = \frac{1}{(s+a)(s+b)} = \begin{cases} \frac{1}{b-a}\left(\frac{1}{s+a} - \frac{1}{s+b}\right) & (\text{if } a \neq b) \\ \frac{1}{(s+a)^2} & (\text{if } a = b) \end{cases}$$

$$\text{hence } \mathcal{L}^{-1}(F(s)) = \begin{cases} \frac{1}{b-a}\left(e^{-at} - e^{-bt}\right) & (\text{if } a \neq b) \\ e^{-at}\, t & (\text{if } a = b) \end{cases}$$

10/10

§6-2 P216.

4. Answer:

Let $Y = \mathcal{L}(y)$, then we obtain: $s^2 Y - s y(0) - y'(0) + 9Y = \frac{10}{s+1}$

$$\Rightarrow (s^2+9)Y = \frac{10}{s+1}$$

$$Y = \frac{10}{(s^2+9)(s+1)}$$

The transfer function is $Q = \frac{1}{s^2+9}$

then $Y = Q\frac{10}{s+1} = \frac{10}{(s^2+9)(s+1)} = a \cdot \frac{3}{s^2+9} + b \cdot \frac{s}{s^2+9} + c \cdot \frac{1}{s+1}$

$$\Rightarrow 3a(s+1) + bs(s+1) + c(s^2+9) = 10$$

$$\Rightarrow \begin{cases} (3a+b)s = 0 \\ (c+b)s^2 = 0 \\ 3a+9c = 10 \end{cases} \Rightarrow \begin{cases} a = \frac{1}{3} \\ b = -1 \\ c = 1 \end{cases}$$

hence $Y = \frac{1}{3}\cdot\frac{3}{s^2+9} - \frac{s}{s^2+9} + \frac{1}{s+1}$

$y(t) = \mathcal{L}^{-1}(Y) = \frac{1}{3}\sin 3t - \cos 3t + e^{-t}$

5. Answer:

Let $Y = \mathcal{L}(y)$, we obtain: $s^2Y - sy(0) - y'(0) - \frac{1}{4}Y = 0$

$\Rightarrow s^2Y - 12s - 0 - \frac{1}{4}Y = 0$

$\Rightarrow (s^2 - \frac{1}{4})Y = 12s$

$\Rightarrow Y = \frac{12s}{s^2 - \frac{1}{4}} = 12\frac{s}{s^2 - (\frac{1}{2})^2} \Rightarrow$

$y(t) = \mathcal{L}^{-1}(Y) = 12\cosh\frac{1}{2}t$

17. Answer:

$f(t) = te^{-at}$ $\quad f'(t) = (1-at)e^{-at}$ $\quad f(0) = 0$

$\mathcal{L}(f') = s\mathcal{L}(f) - f(0) \Rightarrow \mathcal{L}[e^{-at} - ate^{-at}] = s\mathcal{L}(f)$

$\Rightarrow \mathcal{L}(e^{-at}) - a\mathcal{L}(f) = s\mathcal{L}(f)$

$\Rightarrow \mathcal{L}(f) = \frac{1}{s+a}\cdot\frac{1}{s+a} = \frac{1}{(s+a)^2}$

19. Answer:

$f(t) = \sin^2 wt$ $\quad f'(t) = 2\sin wt\cdot\cos wt\cdot w = w\sin 2wt$ $\quad f(0) = 0,\ f'(0) = 0$

$\mathcal{L}(f') = \mathcal{L}(w\sin 2wt) = s\mathcal{L}(f) - f(0) = s\mathcal{L}(f)$

hence $\mathcal{L}(f) = \frac{1}{s}\cdot\frac{2w^2}{s^2+4w^2} = \frac{2w^2}{s(s^2+4w^2)}$ 10/10

26. Answer:

$\mathcal{L}(F) = \frac{1}{s^4 - s^2} = \frac{1}{s}\left(\frac{-1}{s} + \frac{\frac{1}{2}}{s-1} + \frac{\frac{1}{2}}{s+1}\right)$ 10/10

Because $\mathcal{L}^{-1}\left(\frac{-1}{s}+\frac{\frac{1}{2}}{s-1}+\frac{\frac{1}{2}}{s+1}\right) = -1+\frac{1}{2}e^{t}+\frac{1}{2}e^{-t}$

Based on Theorem 3,

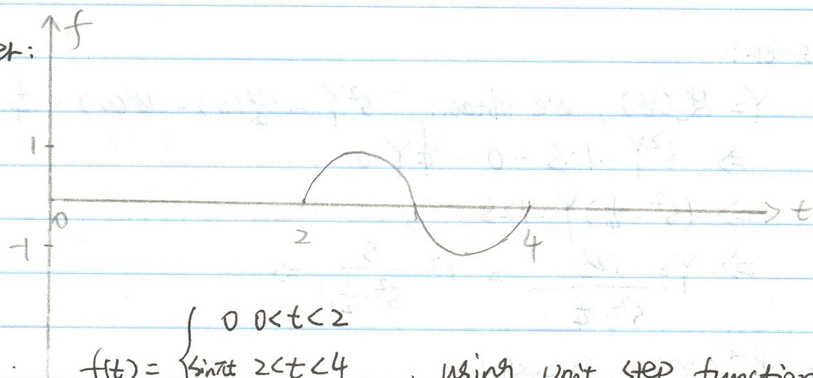
$$\mathcal{L}^{-1}(f) = \int_0^t \left(-1+\tfrac{1}{2}e^{t}+\tfrac{1}{2}e^{-t}\right)d\tau$$

$$= -t+\tfrac{1}{2}(e^{t}-e^{-t})$$

$$= -t+ \sinh t$$

§6.3 P223.

6. Answer:



$$f(t) = \begin{cases} 0 & 0<t<2 \\ \sin\pi t & 2<t<4 \\ 0 & t>4 \end{cases}$$

, using unit step functions

we obtain $f(t) = 0+(\sin\pi t)(u(t-2)-u(t-4))+0$

$= (\sin\pi t)\,u(t-2)-(\sin\pi t)\cdot u(t-4)$

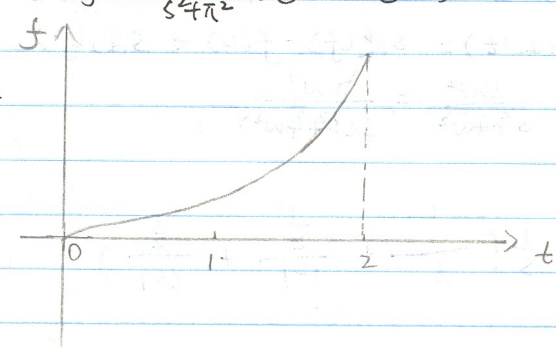
$\cancel{\mathcal{L}(f) = \frac{\pi}{s^2+\pi^2}e^{-2s}+\frac{\pi}{s^2+\pi^2}e^{-4s}}$

$$\mathcal{L}\{(\sin\pi t)\,u(t-2)\} = \mathcal{L}\{\sin\pi(t-2)\cdot u(t-2)\} = \frac{\pi}{s^2+\pi^2}e^{-2s}$$

$$\mathcal{L}\{(\sin\pi t)\,u(t-4)\} = \mathcal{L}\{\sin\pi(t-4)\cdot u(t-4)\} = \frac{\pi}{s^2+\pi^2}e^{-4s}$$

hence, $\mathcal{L}(f) = \frac{\pi}{s^2+\pi^2}(e^{-2s}-e^{-4s})$

10. Answer



$$~~t<1,~~ \; y = \mathcal{L}^{-1}(Y) = \mathcal{L}^{-1}\left(\frac{1}{s^2(s^2+1)} - \frac{s+1}{s^2(s^2+1)} e^{-s} \right)$$

$$= \mathcal{L}^{-1}\left[\frac{1}{s^2} - \frac{1}{s^2+1} + \left(\frac{s}{s^2+1} + \frac{1}{s^2+1} - \frac{1}{s^2} + \frac{1}{s} \right) e^{-s} \right]$$

$$= t - \sin t + u(t-1)\left(\cos(t-1) + \sin(t-1) - t\right)$$

hence, if $0<t<1$ $\quad y = t - \sin t$

if $t>1$ $\quad y = \cos(t-1) + \sin(t-1) - \sin t$

39. Answer:

$$v(t) = \begin{cases} 1000 \; [V] & 0<t<2 \\ 0 & t>2 \end{cases}, \quad v(t) = 1 - u(t-2) \; [kV]$$

$$= 1000\,(1 - u(t-2)) \; [V]$$

According to the given conditions,

$$i' + \frac{1}{0.5}\int_0^t i(\tau)\,d\tau + 2i = 1000\,(1 - u(t-2))$$

Let $I(s) = \mathcal{L}(i)$, then

$$sI - 0 + \frac{2I}{s} + 2I = 1000\left(\frac{1}{s} - \frac{1}{s} e^{-2s} \right)$$

$$\Rightarrow \quad I = 1000\left(\frac{1}{s} - \frac{1}{s} e^{-2s} \right) \Big/ \left(\frac{2}{s} + s + 2 \right)$$

$$= \frac{1000\,(1 - e^{-2s})}{s^2 + 2s + 2}$$

$$i(t) = \mathcal{L}^{-1}(I) = 1000\,\mathcal{L}^{-1}\left[\frac{1}{(s+1)^2+1} - \frac{e^{-2s}}{(s+1)^2+1} \right]$$

$$= 1000\left[e^{-t}\sin t - e^{-(t-2)} \sin(t-2)\, u(t-2) \right]$$

10/10

Mead