p. 230 (3, 10)

3) $y''+4y=\delta(t-\pi)$, $y(0)=8$, $y'(0)=0$

$s^2Y(s)-s\cancel{y(0)}^{8}-\cancel{y'(0)}^{0}+4Y(s)=e^{-\pi s}$

$Y(s)(s^2+4)-8s=e^{-\pi s}$

$Y(s)=\dfrac{e^{-\pi s}}{s^2+4}+8\dfrac{s}{s^2+4} \Rightarrow \mathcal{L}^{-1}\left\{e^{-\pi s}\left(\frac{1}{2}\right)\frac{2}{s^2+2^2}\right\}+8\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}$

$$\boxed{y(t)=8\cos 2t+\tfrac{1}{2}\sin 2t\, u(t-\pi)}$$

10) $y''+5y'+6y=\delta(t-\tfrac{1}{2}\pi)+u(t-\pi)\cos t$, $y(0)=0$, $y'(0)=0$

$s^2Y(s)-s\cancel{y(0)}^{0}-\cancel{y'(0)}^{0}+5[sY(s)-\cancel{y(0)}^{0}]+6Y(s)=e^{-\frac{\pi}{2}s}-e^{-\pi s}\dfrac{s}{s^2+1}$

$(s^2+5s+6)Y(s)=e^{-\frac{\pi}{2}s}-e^{-\pi s}\dfrac{s}{s^2+1}$

$Y(s)=e^{-\frac{\pi}{2}s}\underbrace{\left(\frac{1}{(s+3)(s+2)}\right)}_{(1)}-e^{-\pi s}\underbrace{\left(\frac{s}{(s+3)(s+2)(s^2+1)}\right)}_{(2)}$

$\cos((t-\pi)+\pi)=\cos(t-\pi)\cancel{\cos\pi}^{-1}-\sin(t-\pi)\cancel{\sin\pi}^{0}$

$\cos(t)=-\cos(t-\pi)$

$\mathcal{L}\{-\cos(t-\pi)u(t-\pi)\}$

$-e^{-\pi s}\dfrac{s}{s^2+1}$

(1) $\dfrac{A}{s+3}+\dfrac{B}{s+2}=\dfrac{1}{(s+3)(s+2)}$

$A(s+2)+B(s+3)=1$

$s=-2$ $B=1$

$s=-3$ $A=-1$

$\dfrac{1}{s+2}-\dfrac{1}{s+3}$

(2) $\dfrac{A}{s+3}+\dfrac{B}{s+2}+\dfrac{Cs+D}{s^2+1}=\dfrac{s}{(s+3)(s+2)(s^2+1)}$

$A(s+2)(s^2+1)+B(s+3)(s^2+1)+(Cs+D)(s+3)(s+2)=s$

$\underline{s=-2}$ $B(1)(5)=-2$

$B=-2/5$

$\underline{s=-3}$ $A(-1)(10)=-3$

$A=3/10$

$\underline{s=0}$ $A(2)(1)+B(3)(1)+D(3)(2)=0$

$2A+3B+6D=0$

$3/5-6/5+6D=0$

$6D=3/5$

$D=1/10$

$\underline{s=1}$ $\frac{3}{10}(3)(2)-\frac{2}{5}(4)(2)+(C+\frac{1}{10})(4)(3)=1$

$\frac{18}{10}-\frac{16}{5}+12C+\frac{6}{5}=1$

$\frac{9}{5}-\frac{16}{5}+\frac{6}{5}+12C=\frac{5}{5}$

$12C=\frac{6}{5}$

$C=\frac{1}{10}$

$$\boxed{y(t)=\left[e^{-2(t-\pi/2)}-e^{-3(t-\pi/2)}\right]u(t-\pi/2)-\left[\frac{3}{10}e^{-3(t-\pi)}-\frac{2}{5}e^{-2(t-\pi)}+\frac{1}{10}\cos(t-\pi)+\frac{1}{10}\sin(t-\pi)\right]u(t-\pi)}$$

p. 230 (14ab)

14) a. $\mathcal{L}(f) = \int_0^\infty e^{-st} f(t)\,dt$

$$\int_0^p e^{-s\tau} f(\tau)\,d\tau$$

$$\int_p^{2p} e^{-s\tau} f(\tau)\,d\tau = \int_0^p e^{-s(\tau+p)} f(\tau+p)\,d\tau = e^{-sp}\int_0^p e^{-s\tau} f(\tau)\,d\tau$$

$$\int_{2p}^{3p} e^{-s\tau} f(\tau)\,d\tau = \int_0^p e^{-s(\tau+2p)} f(\tau+2p)\,d\tau = e^{-2sp}\int_0^p e^{-s\tau} f(\tau)\,d\tau$$

$$\mathcal{L}(f) = \left[1 + e^{-sp} + e^{-2sp} + \dots + e^{-nsp}\right] \int_0^p e^{-s\tau} f(\tau)\,d\tau \qquad ?$$

b. $\mathcal{L}(f) = \frac{1}{1-e^{-\frac{2\pi}{\omega}s}} \int_0^{\pi/\omega} e^{-st}\sin\omega t\,dt - \cancel{\int_{\pi/\omega}^{2\pi/\omega} e^{-st}(0)\,dt}^{\,0}$ $\qquad L_s = \int_0^{\pi/\omega} e^{-st}\sin\omega t\,dt$

L_s: $u' = \sin\omega t \quad v = e^{-st}$; $u = \frac{\cos\omega t}{\omega} \quad v' = -se^{-st}$

$$L_s = \frac{\cos\omega t}{\omega} e^{-st}\Big|_0^{\pi/\omega} + \frac{s}{\omega}\int_0^{\pi/\omega} \cos\omega t\, e^{-st}\,dt = L_s = \frac{-e^{-\frac{\pi}{\omega}s}}{\omega} - \frac{1}{\omega} + \frac{s}{\omega}L_c$$

L_c: $u' = \cos\omega t \quad v = e^{-st}$; $u = -\frac{\sin\omega t}{\omega} \quad v' = -se^{-st}$

$$L_c = \frac{-\sin\omega t}{\omega} e^{-st}\Big|_0^{\pi/\omega} - \frac{s}{\omega}\int_0^{\pi/\omega} e^{-st}\sin\omega t\,dt = L_c = \frac{-s}{\omega}L_s$$

$$L_s = \frac{-e^{-\frac{\pi}{\omega}s}}{\omega} - \frac{1}{\omega} + \frac{s}{\omega}\left(\frac{-s}{\omega}L_s\right) \implies L_s\left(1 + \frac{s^2}{\omega^2}\right) = \frac{-e^{-\frac{\pi}{\omega}s}}{\omega} - \frac{1}{\omega}$$

$$L_s = \left(\frac{1}{s^2+\omega^2}\right)\left(-\omega e^{-\frac{\pi}{\omega}s} - \omega\right) = \frac{-\omega\left(1 + e^{-\frac{\pi}{\omega}s}\right)}{s^2+\omega^2}$$

$$\mathcal{L}(f) = \frac{1}{1-e^{-\frac{2\pi}{\omega}s}}\left(\frac{-\omega\left(1+e^{-\frac{\pi}{\omega}s}\right)}{s^2+\omega^2}\right) \implies \boxed{\mathcal{L}(f) = \frac{-\omega\left(1+e^{-\frac{\pi}{\omega}s}\right)}{(s^2+\omega^2)\left(1-e^{-\frac{2\pi}{\omega}s}\right)}}$$

p. 237 (7, 8

7) $t * e^t$ $\quad \int_0^t e^\tau (t-\tau)\,d\tau$ $\quad u = t-\tau \quad v = e^\tau \quad u' = -1 \quad v' = e^\tau$ $\quad (t-\tau)(e^\tau)\Big|_0^t - (-1)\int_0^t e^\tau d\tau$

$$(0-t) + (e^t - 1) = \boxed{e^t - t - 1}$$

8) $y(t) + 4\int_0^t y(\tau)(t-\tau)\,d\tau = 2t = y(t) + 4\left(y(t) * t\right)$

$$Y(s) + 4\left[Y(s)\left(\tfrac{1}{s^2}\right)\right] = \frac{2}{s^2} \implies Y(s)\left(1 + \frac{4}{s^2}\right) = \frac{2}{s^2} \implies Y(s) = \frac{2}{s^2+2^2}$$

$$\boxed{y(t) = \sin 2t}$$

23) $\mathcal{L}(f) = \dfrac{40.5}{s(s^2-9)} = 40.5\left[\left(\frac{1}{s}\right)\left(\frac{1}{s^2-9}\right)\right] \to \mathcal{L}^{-1} \to 40.5\left[1 * \frac{\sinh 3t}{3}\right]$

$$\frac{40.5}{3}\int_0^t \frac{e^{3\tau} - e^{-3\tau}}{2}\,d\tau = \frac{40.5}{6}\left[\int_0^t e^{3\tau} d\tau - \int_0^t e^{-3\tau} d\tau\right] = \frac{40.5}{6}\left[\frac{e^{3\tau}}{3}\Big|_0^t - \frac{e^{-3t}}{-3}\Big|_0^t\right]$$

$$= \frac{40.5}{18}\left[(e^{3t} - 1) + (e^{-3t} - 1)\right] = 2.25\left(e^{3t} + e^{-3t} - 2\right)$$

$$\boxed{y(t) = 4.5(\cosh 3t - 1)}$$