

$$\vec{y}' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \vec{y} \quad 0 = \begin{vmatrix} -\lambda & 1 \\ -1 & -\lambda \end{vmatrix} = \lambda^2 + 1 \Rightarrow \lambda_1 = i, \lambda_2 = -i$$

$$\underline{\lambda_1 = i} \quad -i x_1 + x_2 = 0 \quad \vec{x} = \begin{bmatrix} 1 \\ i \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$e^{it} \vec{x} = \left( \cos t + i \sin t \right) \left( \begin{bmatrix} 1 \\ 0 \end{bmatrix} + i \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$
$$= \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix} + i \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$$

$$\vec{y}(t) = c_1 \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$$

Chapters (7) 8, (4)

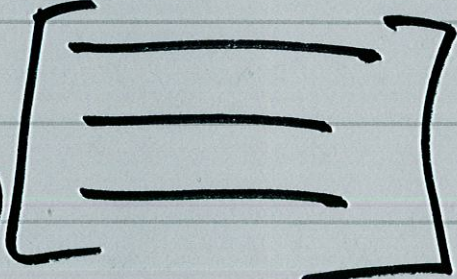
12 problems

$$A_{m \times n} x = b_{m \times 1}$$

$$\underline{[A | b]} \xrightarrow{\text{ERO}}$$

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A rank

l. indep. 

$A_{n \times n}$  |A|

V  $\stackrel{!}{\Rightarrow} v, u \in V$   
 $v + u \stackrel{?}{\in} V$   
 $\stackrel{!}{\Rightarrow} \alpha v \stackrel{?}{\in} V$

vector space

#1  ~~$V = \{ A_{3 \times 3} \mid \det A = 0 \}$~~

$A, B \in V$

$\det(A+B) \stackrel{?}{=} \det A + \det B$

$\det A = 0 \quad \det B = 0$

(ii)  $\left\{ A_{2 \times 2} \mid \begin{array}{l} A \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} A \\ B(C) = (C)B \end{array} \right\} = V$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$(A+B)(C) = (C)(A+B)$

$AC + BC = (CA + CB)A^t = A$

$(\alpha A)C \stackrel{?}{=} C(\alpha A)$

$A, B \in V \Rightarrow A^t = A, B^t = B$

$(\alpha A)^t = \alpha A$

$\Rightarrow \alpha A \in V$

$(A+B)^t = A^t + B^t = (A+B) \Rightarrow A+B \in V$

$$\bullet \underline{\forall A, B \in V}$$

$$\boxed{A^+ = A, B^+ = B}$$
$$\boxed{AC = CA, BC = CB}$$

$$\Rightarrow \underline{A+B} \in V$$

$$\bullet \forall A \in V, \forall \alpha \in R$$

$$\Rightarrow \alpha A \in V$$

$V$  — vector  
space

#2 (i)  $(0, 0, 1)$ ,  ~~$(0, 1, 1)$~~ ,  ~~$(0, 3, 2)$~~

(ii)  $(1, 2, 3)$ ,  $(4, 5, 6)$ ,  $(7, 8, 9)$

(iii)  $(0, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$

(i)  $\det \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 3 & 2 \end{bmatrix}_{3 \times 3} = 0$   $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$  Q. dep.

(ii)  $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{bmatrix}$  Q. dep.

(iii)  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  Q. dep.

#3

$$A = \begin{bmatrix} 2 & -1 \\ 8 & -5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$\left[ \begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 8 & -5 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cc|cc} 2 & -1 & 1 & 0 \\ 0 & +1 & +4 & -1 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{cc|cc} 2/2 & 0 & 5/2 & -1/2 \\ 0 & 1 & 4 & -1 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} 5/2 & -1/2 \\ 4 & -1 \end{bmatrix}$$

(D)

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} -5 & 1 \\ -8 & 2 \end{bmatrix}$$

$$AA^{-1} \stackrel{b_{21}=?}{=} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

#4  ~~$Ax = b$  has no solution~~

$$ax = b \quad \frac{1}{a}$$

$$x = a^{-1}b$$

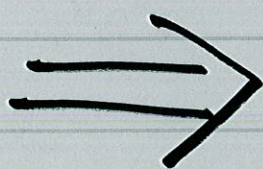
i)  $Ax = 0$  has many solutions  $\Leftrightarrow A^{-1}$  DNE

ii)  $r(A) < n \Leftrightarrow A$  - singular

iii)  $A^{-1}$  DNE

$$x = A^{-1}b$$

$$\left[ \begin{array}{c|c} & \\ \hline 0 & \end{array} \right]$$



$A^{-1}$  DNE