MATH 527 PRACTICE PROBLEMS

1. Which of the following are vector spaces?
   i) The set of all 3x3 matrices A such that $\det A = 0$.
   ii) The set of all 2x2 matrices A such that $A\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} A$.
   iii) The set of all symmetric 3x3 matrices.

   A. iii) only B. i) and ii) C. i) and iii) D. ii) and iii) E. i), ii), and iii)

2. Which of the sets of vectors are linearly independent?
   i) $(0,0,1)$, $(0,1,1)$, $(0,3,2)$ ii) $(1,2,3)$, $(4,5,6)$, $(7,8,9)$
   iii) $(0,0,0)$, $(0,1,0)$, $(0,0,1)$

   A. i) B. ii) C. iii) D. i) and iii) E. None

3. The inverse of the matrix $\begin{pmatrix} 2 & -1 \\ 8 & -5 \end{pmatrix}$ is

   A. $\begin{pmatrix} 5 & -1 \\ 4 & -1 \end{pmatrix}$

   B. $\begin{pmatrix} \frac{5}{2} & 1 \\ 4 & 2 \end{pmatrix}$

   C. $\begin{pmatrix} \frac{5}{2} & -\frac{1}{2} \\ 4 & 1 \end{pmatrix}$

   D. $\begin{pmatrix} \frac{5}{2} & -\frac{1}{2} \\ 4 & -1 \end{pmatrix}$

   E. Matrix has no inverse.

4. Suppose that the system $Ax = b$, where A is an $n \times n$ matrix, has no solutions. Which of the following are true?
   i) The homogeneous equation $Ax = 0$ has infinitely many solutions.
   ii) The rank of A is less than n.
   iii) A has no inverse.

   A. iii) only
   B. i) and ii)
   C. i) and iii)
   D. ii) and iii)
   E. i), ii), and iii)

5. The rank of the matrix $\begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & -3 & -6 & 0 \end{pmatrix}$ is

A. 0
B. 1
C. 2
D. 3
E. 4

6. The eigenvalues for the matrix $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 6 & 3 \end{pmatrix}$ are

A. 1, 2, 3
B. 1, 2, 0
C. 2, 3, 4
D. 1, 3, 0
E. 2, 4, 0

7. The eigenvalues of $\begin{pmatrix} 0 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{pmatrix}$ are 2, 0, and -1. An eigenvector corresponding to -1 is

A. $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ B. $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$
C. $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ D. $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$
E. $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

8. One solution to $y' = \begin{pmatrix} 3 & 0 \\ 1 & 2 \end{pmatrix} y$ is $y = \begin{pmatrix} 0 \\ e^{2t} \end{pmatrix}$. Another linearly independent solution is

A. $\begin{pmatrix} e^{2t} \\ e^{3t} \end{pmatrix}$

B. $\begin{pmatrix} 0 \\ e^{2t} + e^{3t} \end{pmatrix}$

C. $\begin{pmatrix} e^{3t} \\ 0 \end{pmatrix}$

D. $\begin{pmatrix} e^{3t} \\ e^{2t} \end{pmatrix}$

E. $\begin{pmatrix} e^{3t} \\ e^{2t} + e^{3t} \end{pmatrix}$

9. For the system

$$y_1' = y_1 + 3y_2$$
$$y_2' = 4y_1 + 2y_2$$

the origin is

A. an unstable node
B. a stable node
C. a saddle point
D. a stable spiral point
E. an unstable spiral point

10. For the system

$$y_1' = 6y_1 + 9y_2$$
$$y_2' = y_1 + 6y_2$$

the origin is

A. an unstable node
B. a stable node
C. a saddle point
D. a stable spiral point
E. an unstable spiral point

11. For the system

$$dx/dt = \frac{3xy}{1+x^2+y^2} - \frac{1+x^2}{1+y^2}$$

$$dy/dt = x^2 - y^2,$$

the point $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ is

A. an unstable node
B. a stable node
C. a saddle point
D. a stable spiral point
E. an unstable spiral point

12. For the system

$$dx/dt = y$$

$$dy/dt = \sin x,$$

the point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ is

A. an unstable node
B. a stable node
C. a saddle point
D. a stable spiral point
E. an unstable spiral point

13. Assume that a fundamental matrix for the equation $x' = Ax$ is

$$X(t) = \begin{pmatrix} e^{-3t} & e^{-t} \\ -e^{-3t} & e^{-t} \end{pmatrix}.$$

Then the general solution of $x' = Ax + \begin{pmatrix} 2e^{-t} \\ 2 \end{pmatrix}$ is

A. $x(t) = X(t)c + \begin{pmatrix} 2e^{-t} \\ 2 \end{pmatrix}$

B. $x(t) = X(t)c + \begin{pmatrix} e^{2t} - e^{3t} \\ e^t + 1 \end{pmatrix}$

C. $x(t) = X(t)c + \begin{pmatrix} e^{2t} - e^{3t} \\ e^t + t \end{pmatrix}$

D. $x(t) = X(t)c + \begin{pmatrix} \frac{e^{2t}}{2} - \frac{e^{3t}}{3} \\ e^t + t \end{pmatrix}$

E. None of the above.