#14 Given $\mathcal{L}\left\{\frac{e^{-\frac{1}{4t}}}{\sqrt{t}}\right\}^{=f(t)} = \sqrt{\pi}\,\frac{e^{-\sqrt{s}}}{\sqrt{s}}$

$$\mathcal{L}\left\{\frac{e^{-\frac{1}{4t}}}{t^{3/2}}\right\} = \mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^{\infty} \underline{F(\tilde{s})}\,d\tilde{s} = \sqrt{\pi}\int_s^{\infty} \frac{e^{-\sqrt{\tilde{s}}}}{\sqrt{\tilde{s}}}\,d\tilde{s}$$

$u = \sqrt{\tilde{s}}$: $\sqrt{s} \to \infty$

$du = \frac{1}{2}\frac{1}{\sqrt{\tilde{s}}}\,d\tilde{s}$

$$= 2\sqrt{\pi}\int_{\sqrt{s}}^{\infty} e^{-u}\,du = 2\sqrt{\pi}\left[-e^{-u}\right]_{\sqrt{s}}^{\infty}$$

$$= 2\sqrt{\pi}\,e^{-\sqrt{s}}.$$ A

#15 $\mathcal{L}^{-1}\left\{\frac{4}{s^3+4s}\right\} = \mathcal{L}^{-1}\left\{\frac{4}{s(s^2+2^2)}\right\}$

$$\frac{4}{s(s^2+2^2)} = \frac{A}{s} + \frac{Bs+C}{s^2+2^2}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s}{s^2+2^2}\right\}$$

$$= 1 - \cos 2t$$ E.

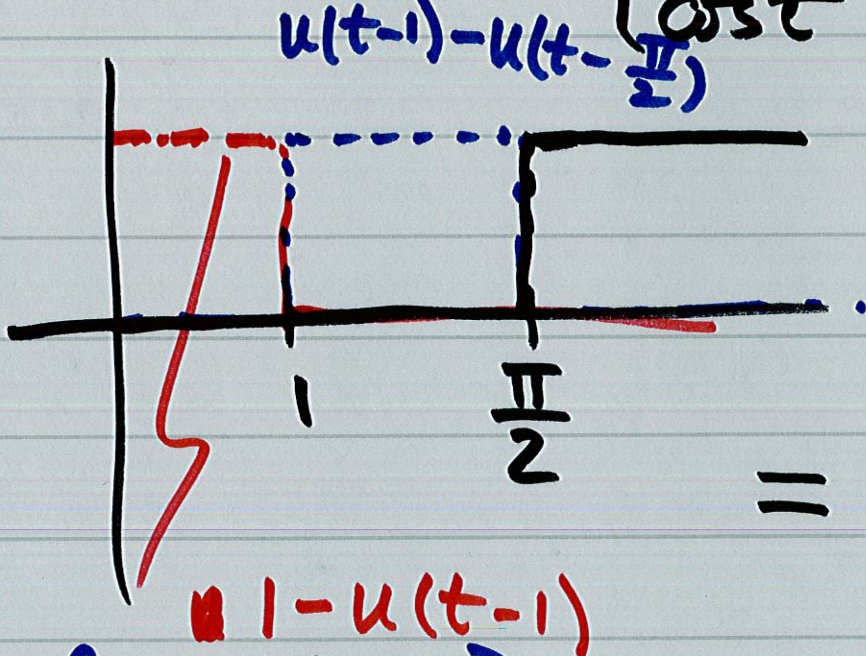
Ex. 1 on P220

$$f(t) = \begin{cases} 2 & 0<t<1 \\ \frac{1}{2}t^2 & 1<t<\frac{\pi}{2} \\ \cos t & t>\frac{\pi}{2} \end{cases} =$$

$$u\left(t-\frac{\pi}{2}\right)\cos t$$

$$+\left(u(t-1) - u\left(t-\frac{\pi}{2}\right)\right)\frac{1}{2}t^2$$

$$+2\left(1 - u(t-1)\right)$$



$$= 2 + u(t-1)\left[\frac{1}{2}t^2 - 2\right] + u\left(t-\frac{\pi}{2}\right)\left[\cos t - \frac{1}{2}t^2\right]$$

$$\frac{1}{2}t^2 - 2 = \frac{1}{2}(t-2)^2 + (t-1) - \frac{3}{2}$$

$$\mathcal{L}\{u(t-a)f(t-a)\} = e^{-as}F(s)$$

$$f(t) = \frac{1}{2}t^2 + t - \frac{3}{2}$$

$$t^2 = [(t-1)+1]^2 = (t-1)^2 + 2(t-1) + 1$$

$$\mathcal{L}\{t\,f(t)\} = -F'(s) \qquad F(s) = \mathcal{L}\{f(t)\}$$

$$\mathcal{L}^{-1}\{F'(s)\} = -t\,f(t) = -t\,\mathcal{L}^{-1}\{F(s)\}$$

$$\underline{\underline{\mathcal{L}^{-1}\{F(s)\}}} = -\frac{1}{t}\,\underline{\underline{\mathcal{L}^{-1}\{F'(s)\}}}$$

$$\mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(\tilde{s})\,d\tilde{s}$$

#19 on P242 §6.6

$\mathcal{L}\{t f(t)\} = -[\mathcal{L}\{f\}]' = -F'(s)$

$\mathcal{L}^{-1}\{-F'(s)\} = tF(s)$

$$\mathcal{L}^{-1}\left\{\ln\frac{s^2+1}{(s-1)^2}\right\} = \mathcal{L}^{-1}\{F(s)\} = -\frac{1}{t}\mathcal{L}^{-1}\{F'(s)\}$$

$$F(s) = \ln\frac{s^2+1}{(s-1)^2} = \ln(s^2+1) - 2\ln(s-1)$$

$$F'(s) = \frac{2s}{s^2+1} - \frac{2}{s-1}, \quad \mathcal{L}^{-1}\{F'(s)\} = 2\cos t - 2e^t$$

$$\mathcal{L}^{-1}\{F(s)\} = -\frac{1}{t}\left(2\cos t - 2e^t\right)$$

#16 $\mathcal{L}^{-1}\left\{\frac{e^{-s}}{s+2}\right\} = u(t-1)\,e^{-2(t-1)}$ $\quad -2(t-1) = -2t+2$

C.

$F(s) = \frac{1}{s+2}$

$$\boxed{\mathcal{L}^{-1}\{e^{-as}F(s)\} = u(t-a)\,f(t-a)}$$

$f(t) = e^{-2t}$

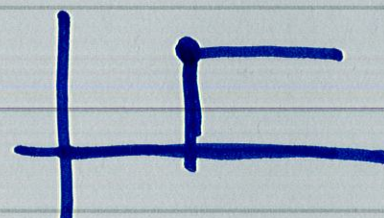
$$\boxed{\mathcal{L}^{-1}\left\{\frac{1}{s-a}\right\} = e^{at}}$$

#17 $\begin{cases} y' + y = u(t-1)\,e^{-2(t-1)} \\ y(0) = 1 \end{cases}$, $y(2) = \cancel{u(1)}\left[e^{-1} - e^{-2}\right] + e^{-2} = e^{-1}$

A

$\cancel{Y(s) = \mathcal{L}\{y(t)\}}$

$F(s) = \mathcal{L}\{e^{-2t}\} = \frac{1}{s+2}$



$$sY - y(0) + Y = e^{-s}\frac{1}{s+2}$$

$$\boxed{Y(s)} = \left(\frac{e^{-s}}{s+2} + 1\right)\Big/ s+1 = e^{-s}\left[\underbrace{\frac{1}{s+1}}_{e^{-t}} - \underbrace{\frac{1}{s+2}}_{e^{-2t}}\right] + \frac{1}{s+1}$$

$$y(t) = u(t-1)\left[e^{-(t-1)} - e^{-2(t-1)}\right] + e^{-t}$$

#18 $\begin{cases} y'' + 2y' + y = \delta(t-1) \\ y(0) = y'(0) = 0 \end{cases}$ $y(2) = e^{-1}$ B

$$s^2 Y - s y(0) - y'(0) + 2\left[sY - y(0) \right] + Y = e^{-s}$$

$$\left(s^2 + 2s + 1\right) Y = e^{-s}$$

$$f(t) = \mathcal{L}^{-1}\left\{ \frac{1}{(s+1)^2} \right\} = t e^{-t}$$

$$Y = e^{-s} \frac{1}{(s+1)^2}$$

$$y(t) = u(t-1) \cdot (t-1) e^{-(t-1)}$$

$$\mathcal{L}^{-1}\left\{ e^{-as} F(s) \right\} = u(t-a) f(t-a)$$

§11.1 – 11.5

$$\boxed{f(x+2\pi)=f(x)}$$

$f(x)$ on $[0, L)$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx, \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$f(x+2L) = f(x)$

$$f(x) = a_0 + \sum_{n=1}^{\infty}\left(a_n \cos \frac{n\pi}{L}x + b_n \sin \frac{n\pi}{L}x\right) \quad b_n = \cdots$$

$$a_0 = \frac{1}{2L}\int_{-L}^{L} f(x)\,dx, \quad a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos \frac{n\pi}{L}x\,dx$$

$f(x)$ on $(0, L)$

F-cosine series

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi}{L} x, \quad a_0 = \frac{1}{L}\int_0^L f(x)\,dx, \quad a_n = \frac{2}{L}\int_0^L f(x) \cos \frac{n\pi}{L} x\,dx$$

F-sine series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{L} x, \quad b_n = \frac{2}{L}\int_0^L f(x) \sin \frac{n\pi}{L} x\,dx$$