Formulas

Let $F(s) = \mathcal{L}\{f\}$ and $G(s) = \mathcal{L}\{g\}$, then

$$\mathcal{L}\{t^n\} = \tfrac{n!}{s^{n+1}}, \qquad \mathcal{L}\{e^{at}\} = \frac{1}{s-a},$$

$$\mathcal{L}\{\sin(bt)\} = \tfrac{b}{s^2+b^2}, \qquad \mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2},$$

$$\mathcal{L}\{e^{at}\sin(bt)\} = \tfrac{b}{(s-a)^2+b^2}, \qquad \mathcal{L}\{e^{at}\cos(bt)\} = \frac{s-a}{(s-a)^2+b^2},$$

$$\mathcal{L}\{f * g\} = \mathcal{L}\{f\}\,\mathcal{L}\{g\} = F(s)\,G(s), \qquad \mathcal{L}\{e^{at}\cos(bt)\} = \frac{s-a}{(s-a)^2+b^2},$$

$$\mathcal{L}\{u(t-c)f(t-c)\} = e^{-cs}\,F(s), \qquad \mathcal{L}\{u(t-c)\} = \frac{e^{-cs}}{s},$$

$$\mathcal{L}\{\delta(t-c)\} = e^{-cs}, \qquad \mathcal{L}\{e^{ct}f(t)\} = F(s-c),$$

$$\mathcal{L}\{tf(t)\} = -F(s), \qquad \mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0),$$

Let $u(r,\,\theta)$ be the Laplace equation in the polar coordinates

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0 \quad \text{in the disk } \Omega = \{(r,\theta)\,|\,0 < r < R,\ -\pi \le \theta < \pi\},$$

then

$$u(r,\,\theta) = a_0 + \sum_{n=1}^{\infty}\left(a_n\left(\frac{r}{R}\right)^n \cos n\theta + b_n\left(\frac{r}{R}\right)^n \sin n\theta\right)$$

with

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta)\,d\theta, \quad a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\,\cos n\theta\,d\theta, \qquad \text{and} \quad b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(\theta)\,\sin n\theta\,d\theta, \quad .$$