Chapter 11 Fourier Analysis

§11.1 Fourier Series

periodic function $f(x+p)=f(x)\ \forall x$

examples

1

x

$\cos x$

$\cos(nx)$

$\sin x$

$\sin(nx)$

Orthogonality

$$\int_{-\pi}^{\pi} \cos nx \cos mx \, dx = \begin{cases} \pi & n = m \\ 0 & n \neq m \end{cases}$$

$$\int_{-\pi}^{\pi} \sin nx \cos mx \, dx = \begin{cases} \pi & n = m \\ 0 & n \neq m \end{cases}$$

$$\int_{-\pi}^{\pi} \cos nx \sin mx \, dx = 0$$

Trigonometric Series

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$

Fourier Series

$$f(x+2x) = f(x) \quad \forall x$$

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

Ex. 2 (Periodic Rectangular Wave)

$$f(x)=\begin{cases} -k & -\pi<x<0 \\ k & 0<x<\pi \end{cases}, \quad f(x+2\pi)=f(x).$$ Find F-S.

Thm 2 Assumptions:

(1) $f(x+2\pi) = f(x)\ \forall x$; (2) f is piecewise continuous in $[-\pi, \pi]$;

(3) at discont. pt x_0, $f'(x_0^-)$ and $f'(x_0^+)$ exist.

$$\Longrightarrow \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right) = \begin{cases} f(x) & \text{at cont. pt.} \\ \dfrac{f(x^-)+f(x^+)}{2} & \text{at discont. pt.} \end{cases}$$