§11.3 Solving ODEs with Fourier Series

Ex. 1 $\quad y' + 2y = f(x) \sim \quad f(x+2\pi) = f(x) \quad \forall x,$

Set $\quad f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}\left(a_n \cos nx + b_n \sin nx\right)$

$$y(x) = \frac{1}{2}c_0 + \sum_{n=1}^{\infty}\left(c_n \cos nx + d_n \sin nx\right) \qquad c_n, d_n - \text{to be determined}$$

Ex. 2 $\quad y'' + 2y' + 2y = f(t), \qquad f(t+2\pi) = f(t)$

$$y(t) = y_h(t) + y_p(t)$$

$$y_h'' + 2y_h' + 2y_h = 0$$

$$y_p'' + 2y_p' + 2y_p = f$$

§11.4 Approximation by Trigonometric Polynomials

$f(x)$ is defined on $(-\pi, \pi)$

$$f(x) \approx F(x) = \frac{1}{2}a_0 + \sum_{n=1}^{N}\left(a_n \cos nx + b_n \sin nx\right)$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx \, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx \, dx$$

general trigonometric poly.

$$G(x) = \frac{1}{2}A_0 + \sum_{n=1}^{N}\left(A_n \cos nx + B_n \sin nx\right)$$

question

$$\min_{G} \| f - G \|^2 = ?$$

where $\|f\|^2 = \int_{-\pi}^{\pi} f^2 \, dx$

Thrm

$$\min_{G} \|f - G\|^2 = \|f - F\|^2 = \|f\|^2 - \|F\|^2$$

Bessel's Inequality

$$\frac{1}{\pi}\|f\|^2 \geq \frac{1}{\pi}\|F\|^2 = \frac{1}{2}a_0^2 + \sum_{n=1}^{N}\left(a_n^2 + b_n^2\right)$$

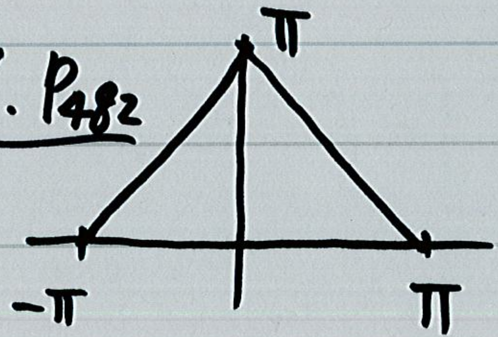
Parseval's Identity

$$\frac{1}{\pi}\|f\|^2 = \frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

#13, P498, §11.4

Prove $1+\frac{1}{3^4}+\frac{1}{5^4}+\frac{1}{7^4}+\cdots=\frac{\pi^4}{96}$

#17. P482



F-series