§11.8 Fourier Cosine and Sine Transforms

Laplace Transform $\quad F(s) = \int_0^\infty e^{-st} f(t)\,dt$

- F-Cosine Transform $\quad A(w) = \frac{2}{\pi}\int_0^\infty f(v)\cos(wv)\,dv$

$$f(x) = \int_0^\infty A(w)\cos(wx)\,dw = \sqrt{\frac{2}{\pi}}\int_0^\infty \left[\sqrt{\frac{2}{\pi}}\int_0^\infty f(y)\cos(wy)\,dy\right]\cos(wx)\,dw$$

$$\mathcal{F}_c(f) = \hat{f}_c(w) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos(wx)\,dx$$

$$f(x) = \mathcal{F}_c^{-1}(f) = \sqrt{\frac{2}{\pi}}\int_0^\infty \hat{f}_c(w)\cos(wx)\,dx$$

- **F-sine transform**

$$\hat{f}_s(w) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin(wx)\, dx = \mathcal{F}_s(f)$$

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty \hat{f}_s(w) \sin(wx)\, dw = \mathcal{F}_s^{-1}(\hat{f}_s)$$

Ex. 1 $f(x) = \begin{cases} k & x \in (0, a) \\ 0 & x > a \end{cases}$

Linearity, Transforms of Derivatives

Assumptions (1) $f(x)$ is absolutely integrable $\iff \int_0^\infty |f(x)|\,dx$ exists

(2) $f(x)$ is p. cont. on every finite interval

• **Linearity** $\mathcal{F}_{c,s}(af + bg) = a\mathcal{F}_{c,s}(f) + b\mathcal{F}_{c,s}(g)$

(3) f' is p. cont. on every finite interval

(4) $\lim\limits_{x\to\infty} f(x) = 0$

• **Derivatives** $\mathcal{F}_c(f'(x)) = w\mathcal{F}_s(f(x)) - \sqrt{\frac{2}{\pi}}\, f(0)$

$\mathcal{F}_s(f'(x)) = -w\mathcal{F}_c(f(x))$

$$\mathcal{F}_c(f'') = -w^2 \mathcal{F}_c(f) - \sqrt{\frac{2}{\pi}}\, f'(0)$$

$$\mathcal{F}_s(f'') = -w^2 \mathcal{F}_s(f) + \sqrt{\frac{2}{\pi}}\, w f(0)$$

<u>Ex. 3</u> $\mathcal{F}_c(e^{-ax}) = ?$ where $a > 0$