§4.5 Qualitative Methods for Nonlinear Systems

$$\vec{y}\,'(t) = \vec{f}(\vec{y}) \qquad \text{autonomous system} \qquad \vec{f}(\vec{y}) = \begin{bmatrix} f_1(y_1, y_2) \\ f_2(y_1, y_2) \end{bmatrix}$$

<u>critical pts</u>: $\vec{y} = (y_1, y_2)$ s.t.

$$\vec{f}(\vec{y}) = \vec{0}$$

<u>Taylor expansion at $\vec{y}_0$</u>

$$\vec{f}(\vec{y}) = \vec{f}(\vec{y}_0) + \nabla\vec{f}(\vec{y}_0)\,(\vec{y} - \vec{y}_0) + \vec{R}(\vec{y})$$

$$\nabla\vec{f}(\vec{y}_0) = \begin{bmatrix} \frac{\partial f_1}{\partial y_1}(\vec{y}_0) & \frac{\partial f_1}{\partial y_2}(\vec{y}_0) \\ \frac{\partial f_2}{\partial y_1}(\vec{y}_0) & \frac{\partial f_2}{\partial y_2}(\vec{y}_0) \end{bmatrix}$$

linearization at critical pt $\vec{y}_0$

$$\vec{f}(\vec{y}) \approx \vec{f}(\vec{y}_0) + \nabla\vec{f}(\vec{y}_0)(\vec{y}-\vec{y}_0) = \nabla\vec{f}(\vec{y}_0)(\vec{y}-\vec{y}_0)$$

$$(\vec{y}-\vec{y}_0)' = \nabla\vec{f}(\vec{y}_0)(\vec{y}-\vec{y}_0)$$

- $\vec{f}$ and $\nabla\vec{f}$ are continuous in the neighborhood of $\vec{y}_0$ and $|\nabla\vec{f}(\vec{y}_0)| \neq 0$

$\Longrightarrow$ $\vec{y}' = \vec{f}(\vec{y})$ and $(\vec{y}-\vec{y}_0)' = \nabla\vec{f}(\vec{y}_0)(\vec{y}-\vec{y}_0)$

have the same kind and stability of the critical pt $\vec{y}_0$.

#5 p159 $\begin{cases} y_1' = y_2 \\ y_2' = -y_1 + \frac{1}{2} y_1^2 \end{cases}$ location and type of all critical pts by linearization

#9, p159 $y'' - 9y + y^3 = 0$ location and type of all critical pts by ODE system, linearization