§4.6 Nonhomogeneous Linear System of ODEs

$$\vec{y}'(t) = A\vec{y}(t) + \vec{g}$$

general solution $\vec{y} = \vec{y}_h + \vec{y}_p$

$\vec{y}_h$ — general solution of homog. system

$\vec{y}_p$ — a particular solution of nonhomog. system

Method of Undetermined Coefficient

$$\vec{y}' = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} \vec{y} + \begin{bmatrix} -6 \\ 2 \end{bmatrix} e^{-2t}$$

$$\vec{y}_h = c_1 e^{-2t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + c_2 e^{-4t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\vec{y}_p = e^{-2t} \vec{v} + t e^{-2t} \vec{u}$$

Method of Variation of Parameters

$$\vec{y}_h = \begin{bmatrix} e^{-2t} & e^{-4t} \\ e^{-2t} & -e^{-4t} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \equiv Y(t)\vec{c}$$

$$\vec{y}_p = Y(t)\vec{u}(t)$$

$$Y'\vec{u} + Y\vec{u}' = AY\vec{u} + \vec{g}$$

$$\xrightarrow{Y' = AY} \quad Y\vec{u}' = \vec{g}$$

$$\vec{u}' = Y^{-1}\vec{g}$$