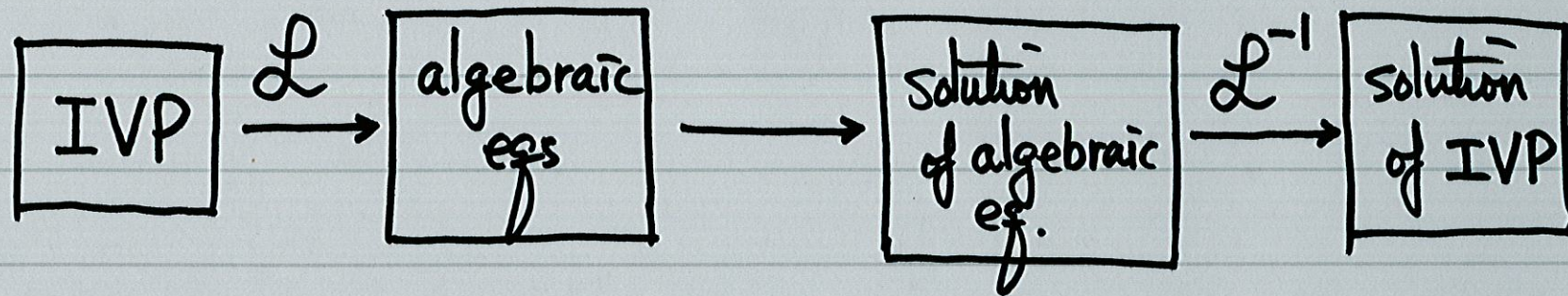


# Chapter 6 Laplace Transform



## §6.1 Laplace Transform. Linearity. 1<sup>st</sup> Shifting Thrm (S-shifting).

$f(t)$  defined  
on  $[0, \infty)$

$$F(s) = \mathcal{L}(f) = \int_0^{\infty} e^{-st} f(t) dt$$

Laplace transform                      kernel

$$f(t) = \mathcal{L}^{-1}(F) \quad \text{inverse Laplace transform}$$

# Table 6.1 $f(t)$ and $\mathcal{L}(f)$

$$f(t) = \mathcal{L}^{-1}(F) \quad 1 \quad t \quad t^2 \quad t^n \quad t^a (a > 0) \quad e^{at}$$

$$F(s) = \mathcal{L}(f) \quad \frac{1}{s} \quad \frac{1}{s^2} \quad \frac{2!}{s^3} \quad \frac{n!}{s^{n+1}} \quad \frac{\Gamma(a+1)}{s^{a+1}} \quad \frac{1}{s-a}$$

$$f(t) = \mathcal{L}^{-1}(F) \quad \cos \omega t \quad \sin \omega t \quad \cosh at \quad \sinh at$$

$$F(s) = \mathcal{L}(f) \quad \frac{s}{s^2 + \omega^2} \quad \frac{\omega}{s^2 + \omega^2} \quad \frac{s}{s^2 - a^2} \quad \frac{a}{s^2 - a^2}$$

$$f(t) = \mathcal{L}^{-1}(F) \quad e^{at} \cos \omega t \quad e^{at} \sin \omega t$$

$$F(s) = \mathcal{L}(f) \quad \frac{s-a}{(s-a)^2 + \omega^2} \quad \frac{\omega}{(s-a)^2 + \omega^2}$$

- $\mathcal{L}(1) =$

- $\mathcal{L}(e^{at}) =$

- $\mathcal{L}(\cosh at) =$

- $\mathcal{L}\{af(t) + bg(t)\} =$

- $\mathcal{L}(t^{n+1})$

s-shifting:  $s \rightarrow s-a$

$$\mathcal{L}\{e^{at}f(t)\} = F(s-a)$$

•  $\mathcal{L}\{e^{at}\cos \omega t\} =$

$$\mathcal{L}\{e^{at}\sin \omega t\} =$$

$$\mathcal{L}^{-1}\left\{\frac{3s-13}{s^2+2s+401}\right\} =$$

•  $f(t)$  is piecewise continuous  $\implies F(s) = \mathcal{L}(f)$  exists  $\forall s > k$   
and  $|f(t)| \leq Me^{kt} \forall t \geq 0$