Chapter 6 Laplace Transform

$$\boxed{\text{IVP}} \xrightarrow{\mathcal{L}} \boxed{\text{algebraic eqs}} \longrightarrow \boxed{\text{solution of algebraic eq.}} \xrightarrow{\mathcal{L}^{-1}} \boxed{\text{solution of IVP}}$$

§6.1 Laplace Transform. Linearity. 1st Shifting Thm (S-shifting).

$f(t)$ defined on $[0, \infty)$

$$\underbrace{F(s) = \mathcal{L}(f)}_{\text{Laplace transform}} = \int_0^\infty \underbrace{e^{-st}}_{\text{kernel}} f(t)\,dt$$

$f(t) = \mathcal{L}^{-1}(F)$ inverse Laplace transform

Table 6.1 $f(t)$ and $\mathcal{L}(f)$

$f(t)=\mathcal{L}^{-1}(F)$	1	t	t^2	t^n	$t^a\ (a>0)$	e^{at}
$F(s)=\mathcal{L}(f)$	$\frac{1}{s}$	$\frac{1}{s^2}$	$\frac{2!}{s^3}$	$\frac{n!}{s^{n+1}}$	$\frac{\Gamma(a+1)}{s^{a+1}}$	$\frac{1}{s-a}$

$f(t)=\mathcal{L}^{-1}(F)$	$\cos\omega t$	$\sin\omega t$	$\cosh at$	$\sinh at$
$F(s)=\mathcal{L}(f)$	$\frac{s}{s^2+\omega^2}$	$\frac{\omega}{s^2+\omega^2}$	$\frac{s}{s^2-a^2}$	$\frac{a}{s^2-a^2}$

$f(t)=\mathcal{L}^{-1}(F)$	$e^{at}\cos\omega t$	$e^{at}\sin\omega t$
$F(s)=\mathcal{L}(f)$	$\frac{s-a}{(s-a)^2+\omega^2}$	$\frac{\omega}{(s-a)^2+\omega^2}$

- $\mathcal{L}(1) =$

- $\mathcal{L}(e^{at}) =$

- $\mathcal{L}(\cosh at) =$

- $\mathcal{L}\{af(t) + bg(t)\} =$

- $\mathcal{L}(t^{n+1})$

<u>s-shifting</u> : $s \longrightarrow s-a$

$$\mathcal{L}\left\{e^{at} f(t)\right\} = F(s-a)$$

- $\mathcal{L}\left\{e^{at} \cos \omega t\right\} =$

  $\mathcal{L}\left\{e^{at} \sin \omega t\right\} =$

  $\mathcal{L}^{-1}\left\{\dfrac{3s-137}{s^2+2s+401}\right\} =$

- $f(t)$ is piecewise continuous and $|f(t)| \leq Me^{kt} \ \forall t \geq 0$ $\Longrightarrow F(s) = \mathcal{L}(f)$ exists $\forall s > k$