§6.2 Transforms of Derivatives and Integrals. ODEs.

- $\mathcal{L}(f') = s\mathcal{L}(f) - f(0)$

  $\mathcal{L}(f'') = s^2\mathcal{L}(f) - sf(0) - f'(0)$

  $\mathcal{L}(f^{(n)}) = s^n\mathcal{L}(f) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$

Ex. $f(t) = t\sin\omega t$, $\mathcal{L}(f'') =$

$\mathcal{L}(t\sin\omega t) = \dfrac{2\omega s}{(s^2+\omega^2)^2}$

Ex. $f(t) = \cos\omega t$

$\mathcal{L}(f'') =$

Laplace Transform of Integral of Function

$$\mathcal{L}\left\{\int_0^t f(\tau)\,d\tau\right\} = \frac{1}{s}\mathcal{L}(f) = \frac{1}{s}F(s)$$

- $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+\omega^2)}\right\} =$

IVP $\begin{cases} y'' + a y' + b y = r(t) \\ y(0) = K_0, \; y'(0) = K_1 \end{cases}$

$$Q(s) = \frac{1}{s^2 + as + b} = \frac{1}{(s+\frac{1}{2}a)^2 + b - \frac{1}{4}a^2}$$

transfer function

$$Y(s) = \left[(s+a)\, y(0) + y'(0) \right] Q(s) + R(s) Q(s) \Longrightarrow y(t) = \mathcal{L}^{-1}(Y)$$

- $\begin{cases} y'' - y = t \\ y(0) = 1, \; y'(0) = 1 \end{cases}$

- (shifted data prob)

$$\begin{cases} y'' + y = 2t \\ y(\frac{1}{4}\pi) = \frac{1}{2}\pi,\ y'(\frac{\pi}{4}) = 2-\sqrt{2} \end{cases} \Longrightarrow \begin{cases} \tilde{y}'' + \tilde{y}' = 2\tilde{t} + \frac{\pi}{2} \\ \tilde{y}(0) = \frac{\pi}{2},\ \tilde{y}'(0) = 2-\sqrt{2} \end{cases}$$

$$t = \tilde{t} + \frac{1}{4}\pi$$

$$\tilde{y}(\tilde{t}) = y(t) \Rightarrow y'(t) = \frac{dy}{dt} = \frac{d\tilde{y}}{d\tilde{t}} \cdot \frac{d\tilde{t}}{dt} = \tilde{y}'(\tilde{t})$$