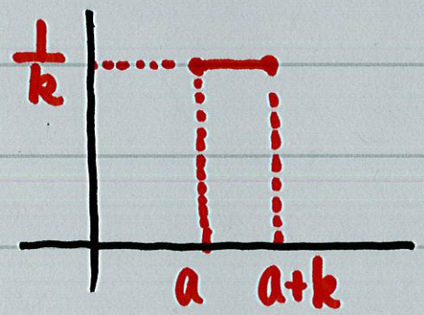
§6.4 Short Impulses. Dirac's Delta Function. Partial Fractions.

$$f_k(t-a) = \begin{cases} \frac{1}{k} & a \le t \le a+k \\ 0 & \text{otherwise} \end{cases}$$



$$I_k = \int_0^\infty f_k(t-a)\,dt = 1$$

- **Dirac Delta Function**

$$\delta(t-a) = \lim_{k \to 0} f_k(t-a) = \begin{cases} \infty & t=a \\ 0 & \text{otherwise} \end{cases}$$

$$\int_0^\infty g(t)\,\delta(t-a)\,dt = g(a), \qquad \mathcal{L}\{\delta(t-a)\} = e^{-as}$$

Ex. 1

$$\begin{cases} y'' + 3y' + 2y = r(t) = u(t-1) - u(t-2) \\ y(0) = 0, \; y'(0) = 0 \end{cases}$$

Ex.2

$$\begin{cases} y'' + 3y' + 2y = \delta(t-1) \\ y(0) = 0, \; y'(0) = 0 \end{cases}$$

Ex. 4 $\begin{cases} y'' + 2y' + 2y = r(t) \\ y(0) = 1, \; y'(0) = -5 \end{cases}$ with $r(t) = \begin{cases} 10 \sin 2t & 0 < t < \pi \\ 0 & t > \pi \end{cases}$.