§6.5 Convolution. Integral Equation.

$$h(t) = (f * g)(t) \equiv \int_0^t f(\tau)\, g(t-\tau)\, d\tau$$

• Properties

$$f * g = g * f \qquad \text{commutative}$$

$$f * (g_1 + g_2) = f * g_1 + f * g_2 \qquad \text{distributive}$$

$$(f * g) * h = f * (g * h) \qquad \text{associative}$$

$$f * 0 = 0 * f = 0$$

$$f * 1 \neq f$$

$$\mathcal{L}\{f*g\} = F(s)G(s) \iff \mathcal{L}^{-1}\{F\cdot G\} = (f*g)(t)$$

Ex. 1 $\mathcal{L}^{-1}\left\{\frac{1}{(s-a)s}\right\} =$

Ex. 2 $\mathcal{L}^{-1}\left\{\frac{1}{(s^2+\omega^2)^2}\right\}$

Ex. 4

$$\begin{cases} y'' + \omega_0^2 y = K \sin \omega_0 t \\ y(0) = y'(0) = 0 \end{cases} \qquad \text{with } \omega_0^2 = \frac{k}{m}$$

Ex. 5

$$\begin{cases} y'' + 3y' + 2y = r \\ y(0) = y'(0) = 0 \end{cases}$$

$$r(t) = \begin{cases} 1 & 1 < t < 2 \\ 0 & t > 2 \end{cases}$$

- **Integral Equation of 2nd kind**

$$y(t) - \int_0^t y(\tau) \sin(t-\tau)\, d\tau = t$$