§6.6 Differentiation and Integration of Transforms. ODEs with Variable Coefficients.

- **Differentiation**

$$F(s) = \int_0^\infty e^{-st} f(t)\,dt \quad \Longrightarrow \quad F'(s) = -\int_0^\infty e^{-st}\, t\, f(t)\,dt = \mathcal{L}\left(t f(t)\right)$$

$\mathcal{L}\{t \sin \beta t\} =$

$\mathcal{L}\{t \cos \beta t\} =$

- **Integration**

$$F(s)=\int_0^{\infty} e^{-st} f(t)\,dt \quad \Longrightarrow \quad \int_s^{\infty} F(\hat{s})\,d\hat{s} = \mathcal{L}\left\{\frac{f(t)}{t}\right\}$$

$$\mathcal{L}^{-1}\left\{\ln\frac{s^2+\omega^2}{s^2}\right\}=$$

$$\mathcal{L}\{ty'\} = -Y(s) - sY'(s), \quad \mathcal{L}\{t^2y''\} = -2sY(s) - s^2Y'(s) + y(0)$$

Ex. $ty'' + (1-t)y' + ny = 0$