§6.7 Systems of ODEs

$$\vec{y}\,'(t) = A\vec{y}(t) + \vec{g}(t)$$

$$\mathcal{L}\{\vec{y}\,'\} = s\vec{Y}(s) - \vec{y}(0)$$

$$\mathcal{L}\{A\vec{y}\} = A\vec{Y}(s)$$

Ex.1 Mixing Problems

$$\vec{y}\,' = \begin{bmatrix} -0.08 & 0.02 \\ 0.08 & -0.08 \end{bmatrix}\vec{y}\,' + \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$\vec{y}(0) = \begin{bmatrix} 0 \\ 150 \end{bmatrix}$$

Ex.3 Two Masses on Springs

$$\begin{cases} y_1'' = -ky_1 + k(y_2 - y_1) \\ y_2'' = -k(y_2 - y_1) - ky_2 \end{cases}$$

$$\vec{y}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \vec{y}'(0) = \begin{bmatrix} \sqrt{3k} \\ -\sqrt{3k} \end{bmatrix}$$

§12.12 Solution of PDEs by Laplace Transforms

- wave equation

$$\frac{\partial^2 w(x,t)}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}, \quad (0,+\infty)\times(0,+\infty)$$

"boundary conditions"

$$\begin{cases} w(0,t) = f(t) \\ \lim\limits_{x\to\infty} w(x,t) = 0 \end{cases}$$

initial conditions

$$\begin{cases} w(x,0) = 0 \\ w_t(x,0) = 0 \end{cases}$$

(1) <u>Laplace transform w.r.t. t</u> $\quad W(x,s) = \mathcal{L}\{w(x,t)\}$

<u>**ODE**</u>

$$\frac{\partial^2 W(x,s)}{\partial x^2} - \frac{s^2}{c^2} W = 0$$

(2) General Solution of ODE

$$W(x,s) = A(s)\, e^{\frac{sx}{c}} + B(s)\, e^{-\frac{sx}{c}}$$

(3) Using BCs to Determine $A(s) = 0$ and $B(s) = F(s)$ $\Longrightarrow$ $W(x,s) = F(s)\, e^{-\frac{sx}{c}}$

(4) Solution of PDE

$$w(x,t) = \mathcal{L}^{-1}\{W(x,s)\}$$