§7.8 Inverse of a Matrix. Gauss-Jordan Elimination.

- $A^{-1}_{n\times n}$ is inverse of $A_{n\times n}$ $\iff$ $AA^{-1} = A^{-1}A = I_{n\times n}$
- $A_{n\times n}$ is singular $\iff$ A has no inverse.
- $A_{n\times n}$ is nonsingular $\iff$ A^{-1} exists $\iff$ $r(A)=n$ $\iff$ $|A| \neq 0$

<u>Computation of A^{-1}</u> by Gauss-Jordan Elimination

$$AA^{-1} = I \xRightarrow{A^{-1}=X=[\vec{x}_1,\cdots,\vec{x}_n]} A[\vec{x}_1,\cdots,\vec{x}_n]=[\vec{e}_1,\cdots,\vec{e}_n]$$

$$[A \,\vdots\, I] \xrightarrow{\text{ERO}} [I \,\vdots\, A^{-1}]$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right]$$

$A \qquad I$

Formulas for Inverse

$$A^{-1} = \frac{1}{|A|}\left[C_{jk}\right]^t = \frac{1}{|A|}\begin{bmatrix} C_{11} & \cdots & C_{n1} \\ C_{12} & \cdots & C_{n2} \\ \vdots & & \vdots \\ C_{1n} & \cdots & C_{nn} \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \quad A^{-1} = \frac{1}{|A|}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad A^{-1} =$$

Properties

- $A = \begin{bmatrix} a_{11} & & 0 \\ & \ddots & \\ 0 & & a_{nn} \end{bmatrix} = \mathrm{diag}(a_{ii}) \qquad A^{-1} = \mathrm{diag}(a_{ii}^{-1})$
- $(A^{-1})^{-1} = A$
- $(AB)^{-1} = B^{-1}A^{-1}$
- $r(A_{n\times n}) = n$ and $A_{n\times n}B_{n\times n} = 0_{n\times n} \Rightarrow B = 0$
- $\det(A_{n\times n}B_{n\times n}) = \det(BA) = \det A \det B$