

MA 174: Multivariable Calculus

Final EXAM (practice)

NAME _____ Class Meeting Time: _____

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

- | | |
|------------------|------------------|
| 1. (5 pts) _____ | 9. (5 pts) _____ |
| 2. (5 pts) _____ | 9. (5 pts) _____ |
| 3. (5 pts) _____ | 9. (5 pts) _____ |
| 4. (5 pts) _____ | 9. (5 pts) _____ |
| 5. (5 pts) _____ | 9. (5 pts) _____ |
| 6. (5 pts) _____ | 9. (5 pts) _____ |

Total Points: _____

1. The arclength of the curve $\vec{r}(t) = \frac{2}{3} t^{3/2} \vec{i} + \frac{2}{3} (2-t)^{3/2} \vec{j} + (t-1) \vec{k}$ for $\frac{1}{4} \leq t \leq \frac{1}{2}$ is:
- A. $\sqrt{2}/4$
 - B. $\sqrt{3}/4$
 - C. $\sqrt{2}/2$
 - D. $3/2$
 - E. $1/2$
2. Find the directional derivative of the function $f(x, y, z) = x^2 y^2 z^6$ at the point $(1, 1, 1)$ in the direction of the vector $\langle 2, 1, -2 \rangle$.
- A. -6
 - B. -2
 - C. 0
 - D. 2
 - E. 6
3. The function $f(x, y) = 3x + 12y - x^3 - y^3$ has
- A. no critical point
 - B. exactly one saddle point
 - C. two saddle points
 - D. two local minimum points
 - E. two local maximum points
4. The function $f(x, y) = x^3 + y^3 - 3xy$ has how many critical points?
- A. None
 - B. One
 - C. Two
 - D. Three
 - E. More than three

5. The max and min values of $f(x, y, z) = xyz$ on the surface $2x^2 + 2y^2 + z^2 = 2$ are

- A. $\pm \frac{\sqrt{2}}{9}$
- B. $\pm \frac{\sqrt{3}}{9}$
- C. $\pm \frac{\sqrt{6}}{9}$
- D. $\pm \frac{2\sqrt{2}}{9}$
- E. $\pm \frac{2\sqrt{3}}{9}$

6. Find the maximum value of $x^2 + y^2$ subject to the constraint $x^2 - 2x + y^2 - 4y = 0$.

- A. 0
- B. 2
- C. 4
- D. 16
- E. 20

7. Find the parametric equations for the line passing through $P = (2, 1, -1)$, and normal to the tangent plane of

$$4x + y^2 + z^3 = 8$$

at P .

- A. $x = t + 4, y = t, z = -t$
- B. $x = 4t + 2, y = 2t + 1, z = 3t - 1$
- C. $\frac{x-2}{4} = \frac{y-1}{2} = \frac{z-1}{3}$
- D. $\frac{x-4}{2} = \frac{y-3}{9} = \frac{2-3}{-1}$
- E. $x = 4t - 2, y = 2t - 1, z = -3t + 1$

8. One vector perpendicular to the plane that is tangent to the surface $2x^2 + xy^2 + z^3 = 2$ at the point $(-1, 1, 1)$ is:

- A. $-3\vec{i} - 2\vec{j} + 3\vec{k}$
- B. $-\vec{i} + \vec{j} + \vec{k}$
- C. $-\vec{i} + 5\vec{k}$
- D. $2\vec{i} - \vec{j} + \vec{k}$
- E. $5\vec{i} + 2\vec{j} + 3\vec{k}$

9. Suppose $z = f(x, y)$, where $x = e^t$ and $y = t^2 + 3t + 2$. Given that $\frac{\partial z}{\partial x} = 2xy^2 - y$ and $\frac{\partial z}{\partial y} = 2x^2y - x$, find $\frac{dz}{dt}$ when $t = 0$.
- A. 3
 B. 6
 C. 15
 D. 9
 E. -1
10. Find the equation in spherical coordinates for $x^2 + y^2 = x$.
- A. $\rho = \sin \phi \cos \theta$
 B. $\rho \sin \phi = \sin^2 \phi \cos \theta$
 C. $\rho = \sin \phi \cos \phi$
 D. $\rho^2 = \rho \cos \phi$
 E. $\rho^2 \sin^2 \phi = \rho \sin \phi \cos \theta$
11. Let $S: x = u - v, y = uv, z = u + v^2$. If $(0, b, 5)$ is a point on the tangent plane to S at $(0, 1, 2)$ on S , then $b =$
- A. 3
 B. 1
 C. -2
 D. 0
 E. 2
12. Find the area of the region bounded by $x = y - y^2$ and $x + y = 0$
- A. $1/3$
 B. $2/3$
 C. 1
 D. $4/3$
 E. $5/3$

13. Find the area in the plane that lies inside the curve $r = 1 + \cos \theta$ and outside the circle $r = 1$.

- A. $\pi/2$
- B. $1 + \pi/2$
- C. $1 + \pi/4$
- D. $2 + \pi/2$
- E. $2 + \pi/4$

14. A sheet of metal occupies the region bounded by the x -axis and the parabola $y = 1 - x^2$. At each point, the density is equal to the distance from the y -axis. Find the mass of the sheet.

- A. $1/4$
- B. $1/3$
- C. $1/2$
- D. $2/3$
- E. 1

15. Evaluate $\int_C ydx + xdy + 2zdz$, where

$$C: F(t) = t(t-1)e^{\sqrt{t}} \vec{i} + \sin\left(\frac{\pi}{2} t^2\right) \vec{j} + \frac{t}{t^2+1} \vec{k}, \quad 0 \leq t \leq 1.$$

- A. 1
- B. $\frac{1}{2}$
- C. $\frac{1}{4}$
- D. 0
- E. -1

16. Let C be the boundary of the triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$ oriented counterclockwise. Then $\int_C ydx - xdy =$

- A. -1
- B. 0
- C. $\frac{1}{2}$
- D. $-\frac{1}{2}$
- E. 2

17. Let $\vec{F} = \nabla f$, $f = \sqrt{x^2 + y^2}$. If C is any smooth curve joining the points $(1, 1)$, $(2, 2)$, then $\int_C 2\vec{F} \cdot d\vec{r} =$

A. $\sqrt{2}$
 B. $\sqrt{12}$
 C. $-\sqrt{2}$
 D. 1
 E. $2\sqrt{2}$

18. Let D be the solid region bounded by the surfaces $x^2 + z^2 = 4$, $y = 1$, $y = 0$, and S be the boundary of D . If $\vec{F}(x, y, z) = \frac{1}{3} (x^3 \vec{i} + y^3 \vec{j} + z^3 \vec{k})$, then with $\vec{n}$ being the unit outward normal, evaluate $\iint_S \vec{F} \cdot \vec{n} d\sigma$.

A. 8π
 B. $\frac{28}{3} \pi$
 C. 28π
 D. 10π
 E. 20

19. Find a, b in the following formula which connect the triple integral from rectangular coordinates to spherical coordinate

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^{\sqrt{x^2+y^2}} y dz dy dx = \int_0^{\pi/2} \int_a^{\pi/2} \int_0^{3 \csc \varphi} b \rho d\rho d\varphi d\theta.$$

A. $a = 0$, $b = \rho^2 \sin \varphi$
 B. $a = \pi/4$, $b = \rho^3 \sin \varphi \sin \theta$
 C. $a = \pi/4$, $b = \rho^3 \sin^2 \varphi \sin \theta$
 D. $a = \frac{\pi}{3}$, $b = \rho^3 \sin^2 \varphi \sin \theta$
 E. $a = -\pi/2$, $b = \rho^3 \sin^2 \varphi$

20. $\vec{F} = 2xy\vec{i} + (x^2 + 3y^2)\vec{j}$ is a conservative vector field, i.e., $\vec{F} = \nabla f$. If $f(0, 0) = 0$, then $f(1, 1) =$

A. 1
 B. 2
 C. 3
 D. 2
 E. 4

21. Evaluate $\iint_S y dS$, where S is the part of the plane $x + 2y + z = 1$ in the 1st octant.

A. $\frac{1}{2\sqrt{6}}$

B. $\frac{1}{2}$

C. $\boxed{\frac{\sqrt{6}}{24}}$

D. $\sqrt{5}$

E. $\frac{\sqrt{5}}{24}$

22. If $\vec{F}(x, y, z) = xz\vec{i} + xyz\vec{j} - y^2\vec{k}$, then $\text{curl } \vec{F}$ evaluated at $(1, 1, 1)$ equals

A. $3\vec{i} - \vec{j} + \vec{k}$

B. $3\vec{i} + \vec{j} - \vec{k}$

C. $\vec{i} + \vec{j} - \vec{k}$

D. $\boxed{-3\vec{i} + \vec{j} + \vec{k}}$

E. $\vec{i} - \vec{j} + 2\vec{k}$

23. Evaluate $\int_0^2 \int_x^2 e^{y^2} dy dx$.

A. $2(e^4 - 1)$

B. $e^4 - 1$

C. $\frac{e^4}{2}$

D. $\boxed{\frac{e^4 - 1}{2}}$

E. $e^4 + 1$

24. Let R be the region in the xy -plane bounded by $y = x$, $y = -x$ and $y = \sqrt{4 - x^2}$. Evaluate the integral

$$\iint_R y dA.$$

A. $\frac{8\sqrt{3}}{2}$

B. $\frac{8}{3\sqrt{2}}$

C. $\frac{4}{\sqrt{2}}$

D. $\boxed{\frac{8\sqrt{2}}{3}}$

E. $4\sqrt{2}$

25. Find the surface area of the part of the surface $z = x^2 + y^2$ below the plane $z = 9$.

A. $\frac{\pi}{4}(3\sqrt{3} - 1)$

B. $\frac{\pi}{4}(3\sqrt{3} - 2\sqrt{2})$

C. $\boxed{\frac{\pi}{6}(37^{3/2} - 1)}$

D. $\frac{\pi}{6}(29^{3/2} - 1)$

E. $\frac{\pi}{6}(27^{3/2} - 1)$

26. Find a, b such that

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^2 z^2 x dz dy dx = \int_0^2 \int_0^a \int_0^b z^2 x dx dy dz.$$

A. $a = 3, b = x$

B. $a = \sqrt{9 - z^2}, b = 3$

C. $\boxed{a = 3, b = \sqrt{9 - y^2}}$

D. $a = z, b = 3$

E. $a = 3, b = \sqrt{9 - x^2}$

27. If $\vec{F}(x, y, z) = (x \sin x + y)\vec{i} + xy\vec{j} + (yz + x)\vec{k}$, then $\text{curl } \vec{F}$ evaluated at $(\pi, 0, 2)$ equals

- A. $\pi\vec{i} - \vec{j} + \vec{k}$
- B. $2\vec{i} - \vec{j} - \vec{k}$
- C. $2\vec{i} - \pi\vec{j} + \vec{k}$
- D. $2\vec{i} - \vec{j} + \pi\vec{k}$
- E. $2\vec{i} + \vec{j} + \vec{k}$

28. Evaluate $\int_C (2x + yz)dx + (2y + xz)dy + xydz$

where $c: \vec{r}(t) = t^2(1+t)\vec{i} + \cos\left(\frac{\pi}{2}t^2\right)\vec{j} + \frac{t^2+1}{t^4+1}\vec{k}$, $0 \leq t \leq 1$.

- A. 1
- B. 2
- C. 3
- D. 4
- E. 5

29. Evaluate $\iint_S (x^2 + y^2 + z^2)dS$ where S is the upper hemisphere of $x^2 + y^2 + z^2 = 2$.

- A. 12π
- B. 8π
- C. 6π
- D. 4π
- E. 3π

30. Evaluate $\int_C -\frac{2y}{x^2 + y^2} dx + \frac{2x}{x^2 + y^2} dy$ where C is the circle $x^2 + y^2 = 1$ oriented counterclockwise.

A. 2π

B. 4π , No to Green's theorem because the function is not continuous at origin

C. 0

D. -4π

E. -2π

31. Calculate the surface integral $\iint_S \vec{F} \cdot \vec{n} dS$ where S is the sphere $x^2 + y^2 + z^2 = 2$ oriented by the outward normal and $\vec{F}(x, y, z) = 5x^3\vec{i} + 5y^3\vec{j} + 5z^3\vec{k}$.

A. $48\sqrt{2}\pi$

B. 16π

C. 24π

D. $25\sqrt{2}\pi$

E. 20π

32. What are the spherical coordinates $(\rho, \varphi, \theta) = \underline{\hspace{2cm}}$ and the cylindrical coordinates $(r, \theta, z) = \underline{\hspace{2cm}}$ for the point $(x, y, z) = (1, 1, 1)$?

Answer: $(\rho, \varphi, \theta) = (\sqrt{3}, \cos^{-1}(\frac{1}{\sqrt{3}}), \frac{\pi}{4})$

Answer: $(r, \theta, z) = (\sqrt{2}, \frac{\pi}{4}, 1)$