MA 174: Multivariable Calculus

Final EXAM (practice)

NAME Solution Class Meeting Time: ____________

NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

Points awarded

1. (5 pts) ________	9. (5 pts) ________
2. (5 pts) ________	9. (5 pts) ________
3. (5 pts) ________	9. (5 pts) ________
4. (5 pts) ________	9. (5 pts) ________
5. (5 pts) ________	9. (5 pts) ________
6. (5 pts) ________	9. (5 pts) ________

Total Points: ________

1. The arclength of the curve $\vec{r}(t) = \frac{2}{3}\, t^{3/2}\vec{i} + \frac{2}{3}(2-t)^{3/2}\vec{j} + (t-1)\vec{k}$ for $\frac{1}{4} \le t \le \frac{1}{2}$ is:

A. $\sqrt{2}/4$

B. $\boxed{\sqrt{3}/4}$

C. $\sqrt{2}/2$

D. $3/2$

E. $1/2$

$\vec{v}(t) = t^{\frac{1}{2}}\vec{i} - (2-t)^{\frac{1}{2}}\vec{j} + \vec{k}$

$|\vec{v}(t)| = \sqrt{t + 2 - t + 1} = \sqrt{3}$

Arc length $= \int_{\frac{1}{4}}^{\frac{1}{2}} |\vec{v}(t)|\, dt = \sqrt{3} \times (\frac{1}{2} - \frac{1}{4}) = \frac{\sqrt{3}}{4}$

2. Find the directional derivative of the function $f(x, y, z) = x^2y^2z^6$ at the point $(1, 1, 1)$ in the direction of the vector $\langle 2, 1, -2\rangle$.

A. -6

B. $\boxed{-2}$

C. 0

D. 2

E. 6

$\vec{\nabla} f = \langle 2xy^2z^6,\ 2x^2yz^6,\ 6x^2y^2z^5 \rangle$

$\vec{\nabla} f \big|_{(1,1,1)} = \langle 2, 2, 6 \rangle$

$\vec{u} = \frac{\langle 2, 1, -2\rangle}{|\langle 2, 1, -2\rangle|} = \langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \rangle$

$D_u f \big|_{\langle 1,1,1\rangle} = \langle 2, 2, 6\rangle \cdot \langle \frac{2}{3}, \frac{1}{3}, -\frac{2}{3} \rangle = -2$

3. The function $f(x, y) = 3x + 12y - x^3 - y^3$ has

A. no critical point

B. exactly one saddle point

C. $\boxed{\text{two saddle points}}$

D. two local minimum points

E. two local maximum points

$f_x = 3 - 3x^2 = 0 \Rightarrow x = \pm 1$

$f_y = 12 - 3y^2 = 0 \Rightarrow y = \pm 2$

critical points $(1, 2)$, $(1, -2)$, $(-1, 2)$ & $(-1, -2)$

$f_{xx} = -6x \qquad f_{yy} = -6y \qquad f_{xy} = 0$

$H = f_{xx} f_{yy} - f_{xy}^2 = 36xy$

$H(1, -2) = -72 < 0, \quad H(-1, 2) = -72 < 0$

$\boxed{(1, -2) \ \&\ (-1, 2) \text{ are saddle points}}$

4. The function $f(x, y) = x^3 + y^3 - 3xy$ has how many critical points?

A. None

B. One

C. $\boxed{\text{Two}}$

D. Three

E. More than three

$\left.\begin{array}{l} f_x = 3x^2 - 3y = 0 \Rightarrow y = x^2 \\ f_y = 3y^2 - 3x = 0 \Rightarrow y^2 = x \end{array}\right\}$

$\Rightarrow (x^2)^2 = x \Rightarrow x(x^3 - 1) = 0 \Rightarrow x = 0$ or $x = 1$

two critical points $(0, 0)$ & $(1, 1)$

5. **The max and min values of $f(x,y,z) = xyz$ on the surface $2x^2 + 2y^2 + z^2 = 2$ are**

A. $\pm\dfrac{\sqrt{2}}{9}$

B. $\pm\dfrac{\sqrt{3}}{9}$

C. $\boxed{\pm\dfrac{\sqrt{6}}{9}}$

D. $\pm\dfrac{2\sqrt{2}}{9}$

E. $\pm\dfrac{2\sqrt{3}}{9}$

$\vec{\nabla} f = \lambda \vec{\nabla} g \Rightarrow \left.\begin{matrix} yz = 4\lambda x \\ xz = 4\lambda y \end{matrix}\right\}$ subtract $\Rightarrow (z-4\lambda)(y-x)=0$

$xy = 2\lambda z$

Case I $z = 4\lambda$

$yz = 4\lambda x = zx$

$\Rightarrow z(x-y)=0$

$\Rightarrow$ (i) $z=0 \Rightarrow f=0$

(ii) $x=y$

$\left.\begin{matrix} x=y \\ xy = 2\lambda z = \frac{1}{2}z^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} z^2 = 2x^2 = 2y^2 \\ 2x^2+2y^2+z^2=2 \end{matrix}\right\} \Rightarrow x = \pm\frac{1}{\sqrt{3}} = y,\ z = \pm\sqrt{\frac{2}{3}} \Rightarrow f = \pm\frac{\sqrt{6}}{9}$

Case II $y=x$

$yz = 4\lambda x = 4\lambda y \Rightarrow$ (i) $y=0 \Rightarrow f=0$

(ii) $z = 4\lambda$ (same as Case I (ii))

$\Rightarrow f = \pm\frac{\sqrt{6}}{9}$

6. **Find the maximum value of x^2+y^2 subject to the constraint $x^2-2x+y^2-4y=0$.**

A. 0

B. 2

C. 4

D. 16

E. $\boxed{20}$

$\vec{\nabla} f = \lambda\vec{\nabla} g \Rightarrow \left.\begin{matrix} 2x = (2x-2)\lambda \Rightarrow 4x = 4x\lambda - 4\lambda \\ 2y = (2y-4)\lambda \Rightarrow 2y = 2y\lambda - 4\lambda \end{matrix}\right\}$ subtract $\Rightarrow (\lambda-1)(2x-y)=0$

Case I $\lambda = 1$

No solution b/c

$2x = 2x-2$

Case II $\left.\begin{matrix} y = 2x \\ x^2-2x+y^2-4y=0 \end{matrix}\right\} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases}$ or $\begin{cases} x=2 \\ y=4 \end{cases}$

$\Rightarrow (x^2+y^2)_{max} = 20$

7. **Find the parametric equations for the line passing through $P = (2,1,-1)$, and normal to the tangent plane of**

$$4x + y^2 + z^3 = 8$$

at P.

A. $x = t+4, y = t, z = -t$

B. $\boxed{x = 4t+2, y = 2t+1, z = 3t-1}$

C. $\dfrac{x-2}{4} = \dfrac{y-1}{2} = \dfrac{z-1}{3}$

D. $\dfrac{x-4}{2} = \dfrac{y-3}{9} = \dfrac{2-3}{-1}$

E. $x = 4t-2,\ y = 2t-1,\ z = -3t+1$

$\vec{v} = \vec{\nabla} f = \langle 4, 2y, 3z^2\rangle\Big|_P = \langle 4, 2, 3\rangle$

$\Rightarrow x = 4t+2$

$y = 2t+1$

$z = 3t-1$

8. **One vector perpendicular to the plane that is tangent to the surface $2x^2 + xy^2 + z^3 = 2$ at the point $(-1,1,1)$ is:**

A. $\boxed{-3\vec{i} - 2\vec{j} + 3\vec{k}}$

B. $-\vec{i}\vec{j} + \vec{k}$

C. $-\vec{i}5\vec{k}$

D. $2\vec{-}\vec{j} + \vec{k}$

E. $5\vec{i} + 2\vec{j} + 3\vec{k}$

normal of the tangent plane.

$\vec{\nabla} f = \langle 4x+y^2, 2xy, 3z^2\rangle \Big|_{(-1,1,1)}$

$= \langle -4+1, -2, 3\rangle$

$= \langle -3, -2, 3\rangle$

9. Suppose $z = f(x, y)$, where $x = e^t$ and $y = t^2 + 3t + 2$. Given that $\frac{\partial z}{\partial x} = 2xy^2 - y$ and $\frac{\partial z}{\partial y} = 2x^2y - x$, find $\frac{dz}{dt}$ when $t = 0$.

A. 3

B. 6

C. $\boxed{15}$

D. 9

E. -1

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$$= \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$= (2xy^2 - y)e^t + (2x^2y - x)(2t+3) \Rightarrow \left.\frac{dz}{dt}\right|_{t=0} = 15$$

$t=0 \quad x = 1 \quad y = 2$

10. Find the equation in spherical coordinates for $x^2 + y^2 = x$.

A. $\rho = \sin\phi\cos\theta$

B. $\rho\sin\phi = \sin^2\phi\cos\theta$

C. $\rho = \sin\phi\cos\phi$

D. $\rho^2 = \rho\cos\phi$

E. $\boxed{\rho^2\sin^2\phi = \rho\sin\phi\cos\theta}$

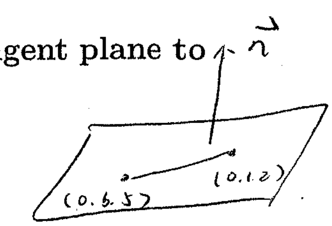
$x = \rho\sin\phi\cos\theta$

$y = \rho\sin\phi\sin\theta$

$\Rightarrow \rho^2\sin^2\phi = \rho\sin\phi\cos\theta$

$\Rightarrow \rho\sin\phi = \cos\theta$

11. Let S: $x = u - v$, $y = uv$, $z = u + v^2$. If $(0, b, 5)$ is a point on the tangent plane to S at $(0, 1, 2)$ on S, then $b =$



A. $\boxed{3}$

B. 1

C. -2

D. 0

E. 2

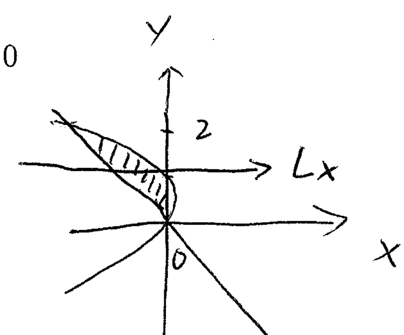
$$\vec{n} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|} = \frac{\vec{i} - 3\vec{j} + 2\vec{k}}{\sqrt{1^2 + (-3)^2 + 2^2}}$$

b/c $\vec{r}_u = \langle 1, v, 1\rangle$ & $u = v = 1$ at $(0, 1, 2)$

$\vec{r}_v = \langle -1, u, 2v\rangle$

$\langle 0, b-1, 3\rangle \cdot \vec{n} = 0 \Rightarrow b = 3$

12. Find the area of the region bounded by $x = y - y^2$ and $x + y = 0$



A. 1/3

B. 2/.3

C. 1

D. $\boxed{4/3}$

E. 5/3

$y - y^2 = -y \Rightarrow y = 0$ or $y = 2$

$x = 0 \quad x = -2$

use x-simple (easier)

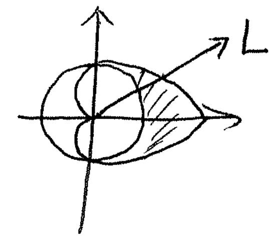
$$A = \int_0^2 \int_{-y}^{y-y^2} dx\,dy = \frac{4}{3}$$

13. Find the area in the plane that lies inside the curve $r = 1+\cos\theta$ and outside the circle $r = 1$.

A. $\pi/2$
B. $1+\pi/2$
C. $1+\pi/4$
D. $2+\pi/2$
E. $\boxed{2+\pi/4}$

$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{1}^{1+\cos\theta} r\,dr\,d\theta$$

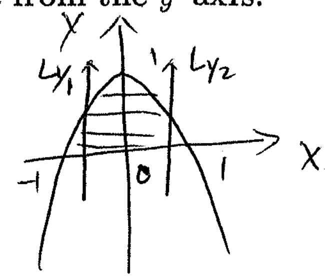
$$= 2 + \frac{\pi}{4}$$



14. A sheet of metal occupies the region bounded by the x–axis and the parabola $y = 1 - x^2$. At each point, the density is equal to the distance from the y–axis. Find the mass of the sheet.

A. 1/4
B. 1/3
C. $\boxed{1/2}$
D. 2/3
E. 1

Divide the region into 2 parts by y-axis b/c density functions are different



$$M = \int_{-1}^{0}\int_{0}^{1-x^2} -x\,dy\,dx + \int_{0}^{1}\int_{0}^{1-x^2} x\,dy\,dx$$

$$= \frac{1}{2}$$

15. Evaluate $\int_C y dx + x dy + 2z dz$, where

$$C\colon F(t) = t(t-1)e^{\sqrt{t}}\,\vec{i} + \sin(\frac{\pi}{2}\,t^2)\vec{j} + \frac{t}{t^2+1}\,\vec{k}, \quad 0 \le t \le 1.$$

A. 1
B. $\frac{1}{2}$
C. $\boxed{\frac{1}{4}}$
D. 0
E. -1

$M = y$	$\otimes$	1	0
$N = x$	1	$\otimes$	0
$P = 2z$	0	0	$\otimes$

$\int_C ydx + xdy + 2zdz$ is exact

$f(x,y,z) = xy + z^2 + C$

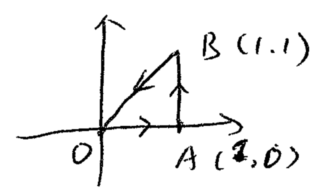
for C: at t = 0 $(x,y,z) = (0,0,0)$
at t = 1 $(x,y,z) = (0,1,\frac{1}{2})$

Thus $\int_C ydx + xdy + 2zdz$
$= f(0,1,\frac{1}{2}) - f(0,0,0)$
$= (\frac{1}{4} + C) - C = \frac{1}{4}$

16. Let C be the boundary of the triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$ oriented counterclockwise. Then $\int_C y dx - x dy =$

A. $\boxed{-1}$
B. 0
C. $\frac{1}{2}$
D. $-\frac{1}{2}$
E. 2

$M = y$
$N = -x$



$$\int_C M dx + N dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$= -2\iint_R dx\,dy = -2A = -2 * \frac{1}{2}$$

$$= -1$$

(Area of the triangle is $\frac{1}{2}$)

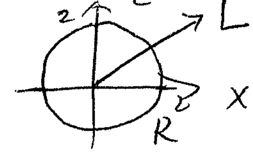
17. Let $\vec{F} = \nabla f$, $f = \sqrt{x^2+y^2}$. If C is any smooth curve joining the points $(1,1)$, $(2,2)$, then $\int_C 2\vec{F}\cdot d\vec{r} =$ $2\int_{(1,1)}^{(2,2)} \vec{F}\cdot d\vec{r}$

A. $\sqrt{2}$
B. $\sqrt{12}$
C. $-\sqrt{2}$
D. 1
E. $\boxed{2\sqrt{2}}$

$= 2\left(f(2,2) - f(1,1)\right)$

$= 2\left(2\sqrt{2} - \sqrt{2}\right) = 2\sqrt{2}$

18. Let D be the solid region bounded by the surfaces $x^2+z^2=4$, $y=1$, $y=0$, and S be the boundary of D. If $\vec{F}(x,y,z) = \frac{1}{3}\left(x^3\,\vec{i} + y^3\vec{j} + z^3\,\vec{k}\right)$, then with $\vec{n}$ being the unit outward normal, evaluate $\iint_S \vec{F}\cdot\vec{n}d\sigma$.



A. 8π
B. $\boxed{\frac{28}{3}\pi}$
C. 28π
D. 10π
E. 20

$\iint_S \vec{F}\cdot\vec{n}\,d\sigma = \iiint_D \vec{\nabla}\cdot\vec{F}\,dV$

$= \iiint_D (x^2+y^2+z^2)\,dV$

$= \iint_R \int_0^1 (x^2+y^2+z^2)\,dy\,dA = \iint_R (x^2+z^2+\frac{1}{3})\,dA$

$= \int_0^{2\pi}\int_0^2 (r^2+\frac{1}{3})\,r\,dr\,d\theta$

$= \frac{28}{3}\pi$

19. Find a, b in the following formula which connect the triple integral from rectangular coordinates to spherical coordinate

$$\int_0^3\int_0^{\sqrt{9-x^2}}\int_0^{\sqrt{x^2+y^2}} y\,dz\,dy\,dx = \int_0^{\pi/2}\int_a^{\pi/2}\int_0^{3\csc\varphi} b\,d\rho\,d\varphi\,d\theta.$$

A. $a=0,\ b=\rho^2\sin\varphi$
B. $a=\pi/4,\ b=\rho^3\sin\varphi\sin\theta$
C. $\boxed{a=\pi/4,\ b=\rho^3\sin^2\varphi\sin\theta}$
D. $a=\frac{\pi}{3},\ b=\rho^3\sin^2\varphi\sin\theta$
E. $a=-\pi/2,\ b=\rho^3\sin^2\varphi$

$b = \rho^2\sin\phi\cdot y = \rho^2\sin\phi \times \rho\sin\phi\sin\theta = \rho^3\sin^2\phi\sin\theta$

$\sqrt{x^2+y^2} = z \Rightarrow \rho\sin\phi = \rho\cos\phi \Rightarrow \phi = \frac{\pi}{4}$

$\Rightarrow a = \frac{\pi}{4}$

20. $\vec{F} = 2xy\vec{i} + (x^2+3y^2)\vec{j}$ is a conservative vector field, i.e., $\vec{F} = \nabla f$. If $f(0,0)=0$, then $f(1,1) =$

A. 1
B. $\boxed{2}$
C. 3
D. 2
E. 4

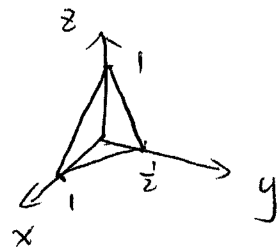
$\frac{\partial f}{\partial x} = 2xy \Rightarrow f(x,y) = x^2y + g(y)$

$\Rightarrow \frac{\partial f}{\partial y} = x^2 + g'(y) = x^2+3y^2 \Rightarrow g(y) = y^3+c$

$\left.\begin{array}{l}\Rightarrow f(x,y) = x^2y+y^3+c \\ f(0,0)=0\end{array}\right\} \Rightarrow c=0$

$\Rightarrow f(x,y) = x^2y+y^3 \Rightarrow f(1,1) = 1+1 = 2$

21. Evaluate $\iint_S y\,dS$, where S is the part of the plane $x + 2y + z = 1$ in the 1st octant.



A. $\dfrac{1}{2\sqrt{6}}$

B. $\dfrac{1}{2}$

C. $\boxed{\dfrac{\sqrt{6}}{24}}$

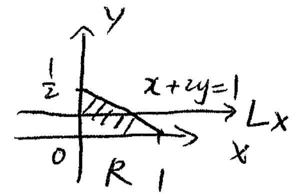
D. $\sqrt{5}$

E. $\dfrac{\sqrt{5}}{24}$

$$\iint_S y\,ds = \iint_R y \cdot \frac{|\vec{\nabla} f|}{|\vec{\nabla} f \cdot \vec{p}\,|}\,dA$$

$\vec{\nabla} f = \langle 1, 2, 1\rangle \qquad \vec{p} = \vec{k} = \langle 0, 0, 1\rangle$

$|\vec{\nabla} f| = \sqrt{6} \qquad |\vec{\nabla} f \cdot \vec{p}\,| = 1$



$$\iint_S y\,ds = \iint_R \sqrt{6}\,y\,dA$$

$$= \int_0^{\frac{1}{2}} \int_0^{1-2y} \sqrt{6}\,y\,dx\,dy$$

$$= \int_0^{\frac{1}{2}} \sqrt{6}\,(y - 2y^2)\,dy = \sqrt{6}\left(\tfrac{1}{2}y^2 - \tfrac{2}{3}y^3\right)\Big]_0^{\frac{1}{2}} = \frac{\sqrt{6}}{24}$$

22. If $\vec{F}(x, y, z) = xz\vec{\imath} + xyz\,\vec{\jmath} - y^2\vec{k}$, then curl $\vec{F}$ evaluated at $(1, 1, 1)$ equals

A. $3\vec{\imath} - \vec{\jmath} + \vec{k}$

B. $3\vec{\imath} + \vec{\jmath} - \vec{k}$

C. $\vec{\imath} + \vec{\jmath} - \vec{k}$

D. $\boxed{-3\vec{\imath} + \vec{\jmath} + \vec{k}}$

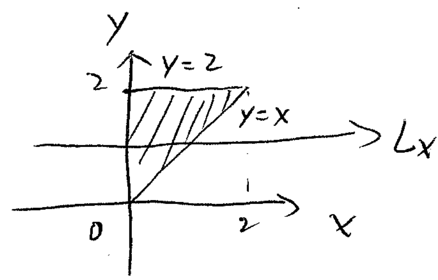
E. $\vec{\imath} - \vec{\jmath} + 2\vec{k}$

$M = xz$	x	0	x
$N = xyz$	yz	x	xy
$P = -y^2$	0	$-2y$	x

$$\text{curl}\,\vec{F} = (-2y - xy)\,\vec{\imath}$$
$$+ (x - 0)\,\vec{\jmath}$$
$$+ (yz - 0)\,\vec{k}$$
$$= (-2y - xy)\,\vec{\imath} + x\,\vec{\jmath} + yz\,\vec{k}$$

$$\text{Curl}\,\vec{F}\,\Big|_{(1,1,1)} = -3\vec{\imath} + \vec{\jmath} + \vec{k}$$

23. Evaluate $\int_0^2 \int_x^2 e^{y^2}\,dy\,dx$.



A. $2(e^4 - 1)$

B. $e^4 - 1$

C. $\dfrac{e^4}{2}$

D. $\boxed{\dfrac{e^4 - 1}{2}}$

E. $e^4 + 1$

↳ reverse the order

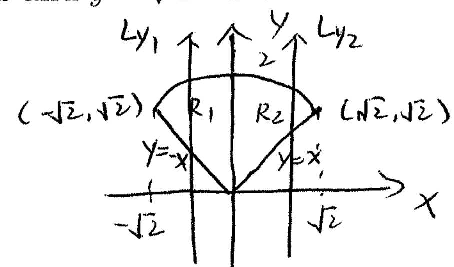
$$\int_0^2 \int_x^2 e^{y^2}\,dy\,dx$$

$$= \int_0^2 \int_0^y e^{y^2}\,dx\,dy$$

$$= \int_0^2 e^{y^2} \cdot y\,dy = \frac{1}{2}e^{y^2}\Big]_0^2 = \frac{e^4 - 1}{2}$$

24. Let R be the region in the xy–plane bounded by $y = x$, $y = -x$ and $y = \sqrt{4 - x^2}$. Evaluate the integral

$$\iint_R y dA.$$



A. $\dfrac{8\sqrt{3}}{2}$

B. $\dfrac{8}{3\sqrt{2}}$

C. $\dfrac{4}{\sqrt{2}}$

D. $\boxed{\dfrac{8\sqrt{2}}{3}}$

E. $4\sqrt{2}$

$$\iint_R y dA = \iint_{R_1} y dA + \iint_{R_2} y dA$$

$$= \int_{-\sqrt{2}}^{0} \int_{-x}^{\sqrt{4-x^2}} y \, dy dx + \int_0^{\sqrt{2}} \int_x^{\sqrt{4-x^2}} y \, dy dx$$

$$= \frac{8}{3}\sqrt{2}$$

25. Find the surface area of the part of the surface $z = x^2 + y^2$ below the plane $z = 9$.

A. $\frac{\pi}{4}(3\sqrt{3} - 1)$

B. $\frac{\pi}{4}(3\sqrt{3} - 2\sqrt{2})$

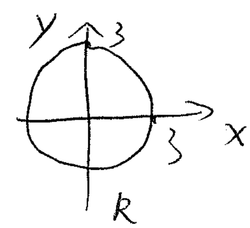
C. $\boxed{\frac{\pi}{6}(37^{3/2} - 1)}$

D. $\frac{\pi}{6}(29^{3/2} - 1)$

E. $\frac{\pi}{6}(2y^{3/2} - 1)$

$f = x^2+y^2-z \Rightarrow \vec{\nabla} f = \langle 2x, 2y, -1\rangle$ $\Rightarrow |\vec{\nabla} f| = \sqrt{4x^2+4y^2+1}$

$\vec{p} = \langle 0,0,1\rangle$ $\quad |\vec{\nabla} f \cdot \vec{p}| = 1$



$$A = \iint_S d\sigma = \iint_R \frac{|\vec{\nabla} f|}{|\vec{\nabla} f \cdot \vec{p}|} dA$$

$$= \iint_R \sqrt{4x^2+4y^2+1} \, dA$$

$$= \int_0^{2\pi} \int_0^3 \sqrt{4r^2+1} \, r dr d\theta = \frac{\pi}{6}(37^{\frac{3}{2}} - 1)$$

26. Find a, b such that

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \int_0^2 z^2 x dz dy dx = \int_0^2 \int_0^a \int_0^b z^2 x \; dx dy dz.$$

A. $a = 3,\ b = x$

B. $a = \sqrt{9 - z^2},\ b = 3$

C. $\boxed{a = 3,\ b = \sqrt{9 - y^2}}$

D. $a = z,\ b = 3$

E. $a = 3,\ b = \sqrt{9 - x^2}$

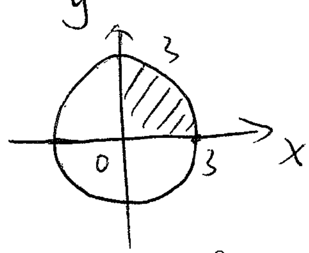
boundary

$0 \le y \le \sqrt{9-x^2} \Rightarrow y^2 + x^2 = 9 \Rightarrow b = \sqrt{9-y^2}$ for x

$\Downarrow \qquad \Uparrow$

$0 \le x \le \sqrt{9-y^2} \Rightarrow a = 3$ for y

limit



Similar to 22

27. If $\vec{F}(x,y,z) = (x\sin x + y)\vec{i} + xy\vec{j} + (yz+x)\vec{k}$, then curl $\vec{F}$ evaluated at $(\pi, 0, 2)$ equals

A. $\pi\vec{i} - \vec{j} + \vec{k}$

B. $\boxed{2\vec{i} - \vec{j} - \vec{k}}$

C. $2\vec{i} - \pi\vec{j} + \vec{k}$

D. $2\vec{i} - \vec{j} + \pi\vec{k}$

E. $2\vec{i} + \vec{j} + \vec{k}$

$M = x\sin x + y$

$N = xy$

$P = yz + x$

$$\begin{array}{ccc} \otimes & 1 & 0 \\ y & \otimes & 0 \\ 1 & z & \otimes \end{array}$$

$\text{curl}\,\vec{F} = z\vec{i} - \vec{j} + (y-1)\vec{k}$ at $(\pi, 0, 2)$

$= 2\vec{i} - \vec{j} - \vec{k}$

Similar to 15

28. Evaluate $\int_C (2x+yz)dx + (2y+xz)dy + xydz$

where c: $\vec{r}(t) = t^2(1+t)\vec{i} + \cos\left(\frac{\pi}{2}t^2\right)\vec{j} + \frac{t^2+1}{t^4+1}\vec{k}$, $0 \le t \le 1$.

A. 1

B. 2

C. $\boxed{3}$

D. 4

E. 5

$M = 2x + yz$

$N = 2y + xz$

$P = xy$

$$\begin{array}{ccc} \otimes & z & y \\ z & \otimes & x \\ y & x & \otimes \end{array}$$

Exact

$f(x,y,z) = x^2 + xyz + y^2 + c$

$t=0 \quad (x,y,z) = (0,1,1)$

$t=1 \quad (x,y,z) = (2,0,1)$

$\int_C Mdx + Ndy + Pdz$

$= f(2,0,1) - f(0,1,1)$

$= (4+c) - (1+c) = 3$

29. Evaluate $\iint_S (x^2+y^2+z^2)dS$ where S is the upper hemisphere of $x^2+y^2+z^2 = 2$.

A. 12π

B. $\boxed{8\pi}$

C. 6π

D. 4π

E. 3π

$x = \sqrt{2}\sin\phi\cos\theta$

$y = \sqrt{2}\sin\phi\sin\theta$

$z = \sqrt{2}\cos\phi$

$0 \le \theta \le 2\pi$

$0 \le \phi \le \frac{\pi}{2}$

$|\vec{r}_\phi \times \vec{r}_\theta| = 2\sin\phi$

$\iint_S (x^2+y^2+z^2)\,dS$

$= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} 2|\vec{r}_\phi \times \vec{r}_\theta|\,d\phi\,d\theta$

$= 8\pi$

30. Evaluate $\int_C -\frac{2y}{x^2+y^2}\,dx + \frac{2x}{x^2+y^2}\,dy$ where C is the circle $x^2+y^2=1$ oriented counterclockwise.

A. 2π

B. $\boxed{4\pi\text{, No to Green's theorem because the function is not continues at origin}}$

C. 0

D. -4π

E. -2π

$\vec{r} = \cos t\,\vec{i} + \sin t\,\vec{j} \qquad 0 \le t \le 2\pi$

$x = \cos t \qquad y = \sin t \qquad dx = -\sin t\,dt \qquad dy = \cos t\,dt$

$\int_C -\frac{2y}{x^2+y^2}\,dx + \frac{2x}{x^2+y^2}\,dy = \int_0^{2\pi} -2\sin t(-\sin t)\,dt + 2\cos t\,(\cos t\,dt)$

$= \int_0^{2\pi} 2\,dt = 4\pi$

31. Calculate the surface integral $\iint_S \vec{F}\cdot\vec{n}\,dS$ where S is the sphere $x^2+y^2+z^2=2$ oriented by the outward normal and $\vec{F}(x,y,z) = 5x^3\vec{i} + 5y^3\vec{j} + 5z^3\vec{k}$.

A. $\boxed{48\sqrt{2}\pi}$

B. 16π

C. 24π

D. $25\sqrt{2}\pi$

E. 20π

$\iint_S \vec{F}\cdot\vec{n}\,ds = \iiint_D \nabla\cdot\vec{F}\,dV = \iiint_D (15x^2+15y^2+15z^2)\,dV$

use spherical coordinates

$x^2+y^2+z^2 = \rho^2$

$dv = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$0 \le \phi \le \pi \qquad 0 \le \theta \le 2\pi$

$0 \le \rho \le \sqrt{2}$

$= \int_0^{2\pi}\int_0^{\pi}\int_0^{\sqrt{2}} 15\rho^2\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$= 48\sqrt{2}\pi$

32. What is the spherical coordinates $(\rho,\varphi,\theta) =$ $(\sqrt{3}, \cos^{-1}\frac{1}{\sqrt{3}}, \frac{\pi}{4})$ and the cylindircal coordinates $(r,\theta,z) =$ $(\sqrt{2}, \frac{\pi}{4}, 1)$ for the point $(x,y,z) = (1,1,1)$?

Answer: $(\rho,\varphi,\theta) = (\sqrt{3}, \cos^{-1}(\frac{1}{\sqrt{3}}), \frac{\pi}{4})$

Answer: $(r,\theta,z) = (\sqrt{2}, \frac{\pi}{4}, 1)$

$\rho = \sqrt{x^2+y^2+z^2} = \sqrt{3}$

$z = \rho\cos\phi = 1$

$\Rightarrow \cos\phi = \frac{1}{\sqrt{3}}$

$\Rightarrow \phi = \cos^{-1}\frac{1}{\sqrt{3}}$

$x = y = 1 \Rightarrow \rho\sin\phi\cos\theta = \rho\sin\phi\sin\theta \Rightarrow \theta = \frac{\pi}{4}$

$r = \sqrt{x^2+y^2} = \sqrt{2}$

$r\cos\theta = r\sin\theta = 1 \Rightarrow \sin\theta = \frac{\sqrt{2}}{2} \Rightarrow \theta = \frac{\pi}{4}$

$z = z = 1$