An elaboration of theory about preventing outbreaks in homogeneous populations to include heterogeneity or preferential mixing

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HIGHLIGHTS

- Population-immunity thresholds are useful only in homogeneous, proportionally-mixing populations.
- Meta-population effective reproduction numbers, $\Re_e$, and related quantities always are useful.
- Heterogeneity in variables affecting sub-population reproduction numbers is relevant.
- Together with preferential mixing among sub-populations, such heterogeneity increases $\Re_e$.
- The partial derivatives of $\Re_e$ with respect to sub-population immunities indicates the optimal strategy.

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ABSTRACT

The goal of many vaccination programs is to attain the population immunity above which pathogens introduced by infectious people (e.g., travelers from endemic areas) will not cause outbreaks. Using a simple meta-population model, we demonstrate that, if sub-populations either differ in characteristics affecting their basic reproduction numbers or if their members mix preferentially, weighted average sub-population immunities cannot be compared with the proportionally-mixing homogeneous population-immunity threshold, as public health practitioners are wont to do. Then we review the effect of heterogeneity in average per capita contact rates on the basic meta-population reproduction number. To the extent that population density affects contacts, for example, rates might differ in urban and rural sub-populations. Other differences among sub-populations in characteristics affecting their basic reproduction numbers would contribute similarly. In agreement with more recent results, we show that heterogeneous preferential mixing among sub-populations increases the basic meta-population reproduction number more than homogeneous preferential mixing does. Next we refine earlier results on the effects of heterogeneity in sub-population immunities and preferential mixing on the effective meta-population reproduction number. Finally, we propose the vector of partial derivatives of this reproduction number with respect to the sub-population immunities as a fundamentally new tool for targeting vaccination efforts.

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1. Introduction

Human populations are heterogeneous, but all differences need not be modeled to answer any specific question. Immunity to vaccine-preventable diseases, for example, is heterogeneous within the United States (Fig. 1). Can differences among and within states be ignored in establishing vaccination coverage targets or monitoring progress in attaining them? The function used routinely for that purpose, the population-immunity threshold, involves the basic reproduction number, denoted $\Re_0$. As this quantity is derived from a mathematical model, ascertaining its adequacy amounts to determining if the model from which it was derived is sufficiently detailed.

Mechanistic models are hypotheses about processes underlying natural phenomena. Simplicity is a virtue because it facilitates their evaluation. But the only way to ensure that one’s model is not too simple is to compare results with those from models that include additional details that might affect them. In transmission modeling, one generally distinguishes sub-populations whose

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