SPATIALLY HETEROGENEOUS INVASION
OF TOXIC PLANT MEDIATED BY HERBIVORY

ZHILAN FENG
Department of Mathematics
Nanjing University of Information Science and Technology
Nanjing 210044, China
and
Department of Mathematics, Purdue University
West Lafayette, IN 47907, USA

WENZHANG HUANG∗
Department of Mathematical Sciences
University of Alabama in Huntsville
Huntsville, AL 35899, USA

DONALD L. DEANGELIS
Department of Biology, University of Miami
Coral Gables, Florida 33124, USA

Abstract. Spatially homogeneous (ODE) and reaction-diffusion models for plant-herbivore interactions with toxin-determined functional response are analyzed. The models include two plant species that have different levels of toxicity. The plant species with a higher level of toxicity is assumed to be less preferred by the herbivore and to have a relatively lower intrinsic growth rate than the less toxic plant species. Two of the equilibrium points of the system representing significant ecological interests are \( E_1 \), in which only the less toxic plant is present, and \( E_2 \), in which the more toxic plant and herbivore coexist while the less toxic plant has gone to extinction. Under certain conditions it is shown that, for the spatially homogeneous system all solutions will converge to the equilibrium \( E_2 \), whereas for the reaction-diffusion model there exist traveling wave solutions connecting \( E_1 \) and \( E_2 \).

1. Introduction. Climate change is apparently already causing latitudinal and altitudinal shifts in ecosystems, as a result of changes in temperature and precipitation over the past few decades, as reviewed by Walther et al. [34]. The upward shift in temperatures in some places is creating conditions favorable to the invasion of vegetation that is adapted to warmer temperatures. Ecotones are places where the effects of climate changes are most likely to be evident [27], and one type of ecotone at which changes in vegetation have been observed both latitudinally and altitudinally is that between woody vegetation and tundra or Alpine sedges, grasses and mosses, where woody shrubs have advanced in places; see [26] for documentation in numerous sites. Woody vegetation, either in prostrate or erect form, may

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∗ The corresponding author.