CONTROL STRATEGIES FOR TB EPIDEMICS*

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Abstract. A model for tuberculosis (TB) that includes immigration of susceptible and infected individuals is presented and analyzed. Infected individuals are structured by time since infection to include a long and variable latency period, and individuals with active TB have an increased mortality rate. Two control problems are formulated and analyzed, minimizing the impact of infection by controlling infected immigrants and/or screening for detection and treatment of infected individuals before they develop active TB.

Key words. tuberculosis, mathematical modeling, control, partial differential equations

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1. Introduction. Tuberculosis (TB) remains a major global health problem, as pointed out in the recent WHO report [34]. In 2013, 9 million people fell ill with TB and 1.5 million died from the disease [35]. TB remains the leading cause of death from an infectious disease.

According to the Millennium Development Goals regarding TB control [34], “about 3 million people who developed TB in 2012 were missed by national notification systems.”

One of the approaches to identify and treat TB-infected individuals is through screening. The main objective of this paper is to identify optimal screening strategies for TB control. Several mathematical models have been developed to study the disease dynamics of TB. For example, optimal treatment strategies for TB have been studied in [24] by applying Pontryagin’s maximum principle, which helped identify the best strategies for case finding (identification of latently infected individuals) and case holding (treatment of individuals with active TB). However, as in many TB models, the model considered in [24] is an ODE system, which assumes an exponentially distributed period of latency. Such an assumption is not realistic for TB, for which the latent period can be very long and variable. In fact, for TB infection only a relatively small proportion of those infected go on to develop the clinical disease [29]. Data from a variety of sources suggest that the lifetime risk of developing clinically evident TB after being infected is approximately 10%, with a 90% likelihood of the infection remaining latent [21]. Individuals who have a latent infection are not clinically ill or capable of transmitting TB [25]. Observations from a study involving 243 tuberculin negative, unvaccinated individuals who developed active TB within 15 years of entering the study, indicate that within each of the six 2.5-year periods, respectively, 28%, 38%, 17%, 10.5%, 4.5%, and 2% developed an active disease [32].

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