Math 262, Practice Midterm 2

1. Determine all the values of a for which the system has no solution.

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 + 2x_3 &= 5 \\ x_1 + 3x_2 + (a^2 - 3)x_3 &= a + 2 \end{aligned}$$

   A. $a = 0$ only.
   B. $a = -2$ only.
   C. $a = -2$ or $a = 2$.
   D. $a = 2$ only.
   E. None of the above.

2. Which of the following set of functions is linearly independent?

   A. $f_1(t) = 1, \quad f_2(t) = t - 3, \quad f_3(t) = t^2 + 2t + 3$
   B. $f_1(t) = 2, \quad f_2(t) = 3t, \quad f_3(t) = 4t - 2$
   C. $f_1(t) = 3, \quad f_2(t) = t^2 + 1, \quad f_3(t) = t^2 - 1$
   D. $f_1(t) = t - 3, \quad f_2(t) = t^2 + 1, \quad f_3(t) = 2t^2 - t, \quad f_4(t) = t^2 + t + 1$
   E. $f_1(t) = t - 3, \quad f_2(t) = t^2 + 1, \quad f_3(t) = t^2 + t - 2$

3. Determine all values of k so that $\{1 + kx^2, 1 + x + x^2, 2 + x\}$ is a basis for $\mathbb{P}_2$.

   A. $k \neq 1$
   B. $k \neq 2$
   C. $k \neq -1$
   D. $k = 1$
   E. $k = -1$

4. The entry of b_{32} of the matrix $A^{-1} = (b_{ij})$ inverse to $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 4 & 1 \\ 2 & -4 & 0 \end{bmatrix}$ is equal to

A. -$\frac{1}{2}$
B. $\frac{3}{2}$
C. $\frac{2}{3}$
D. -1
E. 2

5. Let the vector space $M_n(\mathbb{R})$ be the set of all $n \times n$ matrices with real elements. Which of the following subsets of $M_n(\mathbb{R})$ are the subspace of $M_n(\mathbb{R})$?

(i) The set of all $n \times n$ symmetric matrices.
(ii) The set of all $n \times n$ diagonal matrices.
(iii) The set of all $n \times n$ nonsingular matrices.
(iv) The set of all $n \times n$ singular matrices.

A. (ii) only.
B. (i) and (ii).
C. (ii) and (iii).
D. (i) and (iv).
E. (i) (ii) and (iv).

6. Find a such that $\begin{bmatrix} 5 & -2 \\ 2 & a \end{bmatrix}$ is in the span $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 3 & 2 \end{bmatrix} \right\}$.

A. $a = 4$
B. $a = 6$
C. $a = 8$
D. $a = 10$
E. $a = 12$

7. The homogeneous linear system $Ax = 0$ has only the trivial solution. Which of the following statements must be true?

   A. A is a square matrix and $\det A \neq 0$.

   B. A is a square matrix and $\det A = 0$.

   C. The rank of A equals the number of columns of A.

   D. The rank of A equals the number of rows of A.

   E. The reduced row-echelon form of A is 0.

8. If A and B are 4×4 matrices such that $\det(A) = -3$ and $\det(B) = 2$, then $\det(A(-2B)^{-1}) =$

   A. $\frac{-3}{16}$

   B. $\frac{-3}{4}$

   C. $\frac{3}{4}$

   D. $\frac{-3}{32}$

   E. $\frac{3}{32}$

9. Consider the vectors $v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ $v_2 = \begin{bmatrix} 3 \\ 1 \\ 7 \\ 3 \end{bmatrix}$ $v_3 = \begin{bmatrix} 5 \\ -3 \\ 9 \\ 1 \end{bmatrix}$ $v_4 = \begin{bmatrix} -2 \\ 4 \\ 2 \\ 8 \end{bmatrix}$.
   The dimension of the space $span\{v_1, v_2, v_3, v_4\}$ is equal to

   A. 1

   B. 2

   C. 3

   D. 4

   E. 5

10. Which of the following sets of vectors forms a basis for $\mathbb{R}^3$?

A. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} \right\}$

11. Let S be the subspace of $\mathbb{R}^3$ consisting of all the vectors x of the form $x = (r + s, r - s, 2r + 2s)$, r and s are real. A basis for S is the pair

A. $(1, 1, 2), (1, -1, 2)$

B. $(2, 0, 4), (2, 0, 2)$

C. $(1, 1, 2), (-1, 1, 2)$

D. $(2, 1, 1), (2, -1, 1)$

E. $(1, 1, 2), (2, -1, 1)$

Answers : B, A, C, E, B, B, C, D, C, B, A

12. Let $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & 0 & -3 & 4 \\ 3 & 2 & -5 & 2 \end{bmatrix}$,

(a) Find a basis for the row space of A.

(b) Find a basis for the column space of A.

(c) Find a basis for the null space of A.

(d) Find $rank(A)$ and $dim[nullspace(A)]$.

Answers :

(a) A basis for the row space of A is $\{(1, 1, -1, 1), (0, 1, 2, 3), (0, 0, 0, 1)\}$.

(b) A basis for the column space of A is $\{(1, 1, 3), (1, 0, 2), (1, 4, 2)\}$.

(c) A basis for the null space of A is $\{(3, -2, 1, 0)\}$ (Hint: solve $Ax = 0$).

(d) $rank(A) = 3$ and $dim[nullspace(A)] = 1$.