MATH 262 Quiz 1 January 15

Name: Solution PID: ____________

Solve the problem systematically and neatly and show all your work.

(4pts) 1. Find the equation of the orthogonal trajectories to the curves with equation

$$y^2 = 2x + c.$$

Sol. let the slopes of the tangent lines of $y^2=2x+c$ & its orthogonal trajectories be m_1 & m_2

$$\left.\begin{array}{l} y^2 = 2x+c \Rightarrow 2y\frac{dy}{dx} = 2 \Rightarrow \frac{dy}{dx} = \frac{1}{y} \Rightarrow m_1 = \frac{1}{y} \\ m_1 \times m_2 = -1 \end{array}\right\} \Rightarrow m_2 = -y$$

for orthogonal trajectories. $m_2 = -y \Rightarrow \frac{dy}{dx} = -y$

Solve $\frac{dy}{dx} = -y$

$$\left.\begin{array}{l} ① \ y=0 \longrightarrow \text{equilibrium sol} \\ ② \ y\neq 0 \quad \frac{1}{y}dy = -dx \Rightarrow \ln|y| = -x + C_1 \Rightarrow y = \pm e^{C_1}e^{-x} \end{array}\right\} \Rightarrow \boxed{y = Ke^{-x}}$$

K arbitrary constant

(3pts) 2. Do you think that the initial value problem

$$\frac{dy}{dx} = x\cos(x+y), y(0) = 1$$

has a unique solution? Justify your answer.

Sol.

$$\left.\begin{array}{l} f(x,y) = x\cos(x+y) \\ \Rightarrow \frac{\partial f}{\partial y} = -x\sin(x+y) \end{array}\right\} \text{both } f(x,y) \ \& \ \frac{\partial f}{\partial y} \text{ are continuous functions}$$

Using the Theorem of Existence and Uniqueness, the I.V.P has a unique solution.

(3pts) 3. Verify that

$$y(x) = c_1e^x\cos 2x + c_2e^x\sin 2x$$

is a solution to $y'' - 2y' + 5y = 0$.

Sol. $y(x) = C_1e^x\cos 2x + C_2e^x\sin 2x \Rightarrow y'(x) = C_1e^x\cos 2x - 2C_1e^x\sin 2x + C_2e^x\sin 2x + 2C_2e^x\cos 2x$

$$= (C_1+2C_2)e^x\cos 2x + (C_2-2C_1)e^x\sin 2x$$

$\Rightarrow y''(x) = (C_1+2C_2)e^x\cos 2x - 2(C_1+2C_2)e^x\sin 2x + (C_2-2C_1)e^x\sin 2x + 2(C_2-2C_1)e^x\cos 2x$

$$= (4C_2-3C_1)e^x\cos 2x - (4C_1+3C_2)e^x\sin 2x$$

$$\Rightarrow y''-2y'+5y = [\underbrace{(4C_2-3C_1) - 2(C_1+2C_2) + 5C_1}_{0}]e^x\cos 2x + [\underbrace{-(4C_1+3C_2) - 2(C_2-2C_1) + 5C_2}_{0}]e^x\sin 2x$$

$$= 0 + 0 = 0$$

$\Rightarrow y(x)$ is a solution to $y''-2y'+5y=0$.