MATH 262 Quiz 2 January 22

Name: Solution PID: ______________________

Solve the problem systematically and neatly and show all your work.

(3pts) 1.Solve the initial value problem

$$\frac{dy}{dx} = \frac{2x(y-1)}{x^2+3}, \qquad y(1) = 9.$$

Sol: seperable D.E.

① $y=1$ equilibrium solution

② $y \neq 1$

$$\frac{1}{y-1}\,dy = \frac{2x}{x^2+3}\,dx$$

$$\int \frac{1}{y-1}\,dy = \int \frac{2x}{x^2+3}\,dx + C_1$$

$$\ln|y-1| = \ln(x^2+3) + C_1$$

$$\Rightarrow |y-1| = e^{C_1}(x^2+3)$$

$$\Rightarrow y = 1 + C(x^2+3) \quad (C \neq 0)$$

Combine ① & ②

$y = 1 + C(x^2+3)$ C is arbitrary

$y(1) = 9$

$$\Rightarrow 9 = 1 + C(1^2+3)$$

$$\Rightarrow C = 2$$

Thus

$$\boxed{\begin{aligned} y &= 1 + 2(x^2+3) \\ &= 2x^2+7 \end{aligned}}$$

(3pts) 2. The population in a small village grows at a rate proportional to the number present. Initially the population was 50. After 5 years the population was 800. Find the doubling time.

Sol: $\frac{dp}{dt} = kp$

$$\Rightarrow p = Ce^{kt}, \quad p(0) = 50$$

$$\Rightarrow C = 50$$

$$p = 50e^{kt}, \quad p(5) = 800$$

$$\Rightarrow 800 = 50e^{5k}$$

$$\Rightarrow \frac{800}{50} = e^{5k}$$

$$16 = e^{5k}$$

$$5k = \ln 16$$

$$k = \frac{\ln 16}{5} = \frac{4\ln 2}{5}$$

$$t_d = \frac{\ln 2}{k} = \frac{\ln 2}{\frac{4\ln 2}{5}} = \boxed{\frac{5}{4}}$$

(4pts) 3. Solve the differential equation

$$\frac{dy}{dx} - \frac{2}{x}y = x^2 - 1, \qquad x > 0.$$

Sol: 1^{st}-order linear D.E. $p(x) = -\frac{2}{x}$, $g(x) = x^2-1$

$$I(x) = e^{\int p(x)dx} = e^{\int -\frac{2}{x}dx}$$

$$= e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2} = \frac{1}{x^2}$$

$$\frac{d}{dx}\left(\frac{1}{x^2}y\right) = \frac{1}{x^2}(x^2-1) = 1 - \frac{1}{x^2}$$

$$\frac{1}{x^2}y = \int \left(1 - \frac{1}{x^2}\right)dx + C = x + \frac{1}{x} + C \Rightarrow \boxed{y = x^3 + x + Cx^2}$$