MATH 262 Quiz 4

Name: Solution PID: _____

1. (6pts) Consider the system

$$x_1 + x_2 + x_3 = 2,$$
$$2x_1 + 3x_2 + 2x_3 = 5,$$
$$2x_1 + 3x_2 + (k^2 - 2)x_3 = k + 3.$$

Determine all the values of the constant k for which the above system has

(1) no solution. (2) unique solution. (3) a finite number of solutions.

Sol: $A^\# = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 3 & 2 & 5 \\ 2 & 3 & k^2-2 & k+3 \end{bmatrix} \overset{\substack{A_{12}(-2) \\ A_{13}(-2)}}{\sim} \begin{bmatrix} \boxed{1} & 1 & 1 & 2 \\ 0 & \boxed{1} & 0 & 1 \\ 0 & 1 & k^2-4 & k-1 \end{bmatrix} \overset{A_{23}(-1)}{\sim} \begin{bmatrix} \boxed{1} & 1 & 1 & 2 \\ 0 & \boxed{1} & 0 & 1 \\ 0 & 0 & k^2-4 & k-2 \end{bmatrix}$

① if $k^2-4=0$ & $k-2\neq 0$ then $r=2$, $r^\#=3$, $r\neq r^\#$ $\Rightarrow$ 0 solns $\Rightarrow \boxed{(k=-2)}$

② if $k^2-4\neq 0$, then $r=r^\#=3=n$ $\Rightarrow$ unique soln $\boxed{(k\neq \pm 2)}$

③ if $k^2-4=0$ & $k-2=0$ then $r=r^\#=2<n=3$ $\Rightarrow$ ∞ solns $\Rightarrow \boxed{(k=2)}$

2. (4pts) Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 3 \\ 1 & -1 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix},$$

(1) find A^{-1}.

(2) use A^{-1} to find the solution of $Ax = b$.

Sol: ① $[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 2 & -3 & 3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \overset{\substack{A_{12}(-2) \\ A_{13}(-1)}}{\sim} \left[\begin{array}{ccc|ccc} \boxed{1} & -1 & 2 & 1 & 0 & 0 \\ 0 & -1 & -1 & -2 & 1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right]$

$\overset{M_2(-1)}{\sim} \left[\begin{array}{ccc|ccc} \boxed{1} & -1 & 2 & 1 & 0 & 0 \\ 0 & \boxed{1} & 1 & 2 & -1 & 0 \\ 0 & 0 & \boxed{1} & \frac{1}{2} & 0 & -\frac{1}{2} \end{array}\right] \overset{A_{21}(1)}{\sim} \left[\begin{array}{ccc|ccc} \boxed{1} & 0 & 3 & 3 & -1 & 0 \\ 0 & \boxed{1} & 1 & 2 & -1 & 0 \\ 0 & 0 & \boxed{1} & \frac{1}{2} & 0 & -\frac{1}{2} \end{array}\right]$

$\overset{\substack{A_{32}(1) \\ A_{31}(-3)}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{2} & -1 & \frac{3}{2} \\ 0 & 1 & 0 & \frac{3}{2} & -1 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{2} \end{array}\right] \Rightarrow \boxed{A^{-1} = \begin{bmatrix} \frac{3}{2} & -1 & \frac{3}{2} \\ \frac{3}{2} & -1 & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix}}$

I_3 A^{-1}

② $\vec{x} = A^{-1}\vec{b} = \begin{bmatrix} 4 \\ 1 \\ -1 \end{bmatrix}$