MATH 262 Quiz 5 February 26

Name: Solution PID: ____________________

Solve the problem systematically and neatly and show all your work.

1. (2 pts) If A and B are 5×5 matrices such that $det(A) = -\frac{1}{4}$ and $det(B) = 3$, find $det(-2AB)$).

Sol: $\det(-2AB) = (-2)^5 \det(AB) = -32 \det(A)\det(B) = -32 * (-\frac{1}{4}) * 3 = \boxed{24}$

use properties $\det(KA) = K^n \det(A)$
A — $n\times n$ $\det(AB) = \det(A)\det(B)$
B — $n\times n$

2. (8pts) Let

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & -3 & 3 \\ 1 & -1 & 0 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix},$$

(1) find $det(A)$. (2) use Cramer's rule to solve $Ax = b$.
(3) let $A^{-1} = [b_{ij}]$, find b_{23}. (4) find A^{-1} by using $adj(A)$.

Sol: ① $\det(A) = \begin{vmatrix} 1 & -1 & 2 \\ 2 & -3 & 3 \\ 1 & -1 & 0 \end{vmatrix} \overset{C_{12}(1)}{=\!=} \begin{vmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 1 & 0 & 0 \end{vmatrix} \underset{3^{rd}\text{ row}}{\overset{\text{expand along}}{=\!=}} \begin{vmatrix} 0 & 2 \\ -1 & 3 \end{vmatrix} = \boxed{2}$

② $\det(B_1) = \begin{vmatrix} 1 & -1 & 2 \\ 2 & -3 & 3 \\ 3 & -1 & 0 \end{vmatrix} \overset{C_{21}(3)}{=\!=} \begin{vmatrix} -2 & -1 & 2 \\ -7 & -3 & 3 \\ 0 & -1 & 0 \end{vmatrix} \overset{\text{along 3rd row}}{=\!=} \begin{vmatrix} -2 & 2 \\ -7 & 3 \end{vmatrix} = 8$

$\det(B_2) = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 2 & 3 \\ 1 & 3 & 0 \end{vmatrix} \overset{C_{12}(-1)}{=\!=} \begin{vmatrix} 1 & 0 & 2 \\ 2 & 0 & 3 \\ 1 & 2 & 0 \end{vmatrix} \overset{\text{along 2nd col.}}{=\!=} -2\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -2*(-1) = 2$

$\det(B_3) = \begin{vmatrix} 1 & -1 & 1 \\ 2 & -3 & 2 \\ 1 & -1 & 3 \end{vmatrix} \overset{C_{13}(-1)}{=\!=} \begin{vmatrix} 1 & -1 & 0 \\ 2 & -3 & 0 \\ 1 & -1 & 2 \end{vmatrix} \overset{\text{along 3rd col.}}{=\!=} 2\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix} = 2*(-1) = -2$

$\Rightarrow \boxed{x_1 = \frac{8}{2} = 4 \qquad x_2 = \frac{2}{2} = 1 \qquad x_3 = \frac{-2}{2} = -1}$

③ $b_{23} = \dfrac{C_{32}}{\det(A)} = \dfrac{-\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}}{2} = \boxed{\frac{1}{2}}$

④ $C_{11} = \begin{vmatrix} -3 & 3 \\ -1 & 0 \end{vmatrix} = 3 \qquad C_{12} = -\begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = 3 \qquad C_{13} = \begin{vmatrix} 2 & -3 \\ 1 & -1 \end{vmatrix} = 1$

$C_{21} = -\begin{vmatrix} -1 & 2 \\ -1 & 0 \end{vmatrix} = -2 \qquad C_{22} = \begin{vmatrix} 1 & 2 \\ 1 & 0 \end{vmatrix} = -2 \qquad C_{23} = -\begin{vmatrix} 1 & -1 \\ 1 & -1 \end{vmatrix} = 0$

$C_{31} = \begin{vmatrix} -1 & 2 \\ -3 & 3 \end{vmatrix} = 3 \qquad C_{32} = -\begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 1 \qquad C_{33} = \begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix} = -1$

$$\operatorname{adj}(A) = \begin{bmatrix} C_{11} & C_{21} & C_{31} \\ C_{12} & C_{22} & C_{32} \\ C_{13} & C_{23} & C_{33} \end{bmatrix} = \begin{bmatrix} 3 & -2 & 3 \\ 3 & -2 & 1 \\ 1 & 0 & -1 \end{bmatrix} \Rightarrow A^{-1} = \frac{\operatorname{adj}(A)}{\det(A)} = \boxed{\begin{bmatrix} 3/2 & -1 & 3/2 \\ 3/2 & -1 & 1/2 \\ 1/2 & 0 & -1/2 \end{bmatrix}}$$