MATH 262 Quiz 6

Name: Solution PID: ______________

Solve the problem systematically and neatly and show all your work.

1. (2pts) (a) Find the null space of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$.

   (2pts) (b) Show that nullspace(A) is a subspace of $\mathbb{R}^3$.

Sol: ① nullspace $(A) = \{\vec{x} \in \mathbb{R}^3 : A\vec{x} = \vec{0}\}$

$$A\vec{x} = \vec{0} \Rightarrow \left.\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ 2x_1 + 4x_2 + 6x_3 &= 0 \\ 3x_1 + 6x_2 + 9x_3 &= 0 \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ x_2 &= r \\ x_3 &= s \end{aligned}\right\} \Rightarrow x_1 = -2r - 3s$$

$$\Rightarrow \boxed{\text{nullspace}(A) = \{\vec{x} \in \mathbb{R}^3 : \vec{x} = (-2r-3s, r, s),\ r, s \in \mathbb{R}\}}$$

② $\forall\ \vec{x} \in$ nullspace (A), $\vec{y} \in$ nullspace (A), $k \in \mathbb{R}$

Need to show $\vec{x} + \vec{y} \in$ nullspace (A), $k\vec{x} \in$ nullspace (A)

Let $\vec{x} = (-2r_1 - 3s_1, r_1, s_1)$, $\vec{y} = (-2r_2 - 3s_2, r_2, s_2)$

$$\Rightarrow \boxed{\begin{aligned} &\vec{x} + \vec{y} = (-2(r_1+r_2) - 3(s_1+s_2),\ r_1+r_2,\ s_1+s_2) \in \text{nullspace}(A) \\ &k\vec{x} = (-2kr_1 - 3ks_1,\ kr_1,\ ks_1) \in \text{nullspace}(A) \end{aligned}}$$

2. (2pts) (a) Show that $\mathbf{v}_1 = (2, -1)$, $\mathbf{v}_2 = (3, 2)$ span $\mathbb{R}^2$.

   (2pts) (b) Express the vector $\mathbf{v} = (9, -1)$ as a linear combination of $\mathbf{v}_1$ and $\mathbf{v}_2$.

Sol. ⓐ $\forall\ \vec{v} \in \mathbb{R}^2$ let $\vec{v} = (a, b) = c_1 v_1 + c_2 v_2 = (2c_1 + 3c_2, -c_1 + 2c_2)$

$$\Rightarrow \left.\begin{aligned} 2c_1 + 3c_2 &= a \\ -c_1 + 2c_2 &= b \end{aligned}\right\} \Rightarrow \left.\begin{aligned} c_1 &= \frac{2a - 3b}{7} \\ c_2 &= \frac{a + 2b}{7} \end{aligned}\right\}$$ solution for c_1 & c_2 exist for any a & b

$$\Rightarrow \boxed{\{\vec{v}_1, \vec{v}_2\} \text{ spans } \mathbb{R}^2}$$

OR $\boxed{\text{let } A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \quad \det(A) = 7 \neq 0 \Rightarrow \{\vec{v}_1, \vec{v}_2\} \text{ spans } \mathbb{R}^2}$

ⓑ $\vec{v} = (9, -1) = c_1 \vec{v}_1 + c_2 \vec{v}_2$

$$\Rightarrow \left.\begin{aligned} c_1 &= \frac{2 \times 9 - 3(-1)}{7} = 3 \\ c_2 &= \frac{9 + 2(-1)}{7} = 1 \end{aligned}\right\} \Rightarrow \boxed{\vec{v} = (9, -1) = 3\vec{v}_1 + \vec{v}_2}$$

3. (2pts) Find the value a such that $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ is in the span$\left\{\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 5 & 3 \\ 2 & a \end{bmatrix}\right\}$.

Sol: let $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} = c_1 \begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix} + c_2 \begin{bmatrix} 5 & 3 \\ 2 & a \end{bmatrix} = \begin{bmatrix} 2c_1 + 5c_2 & c_1 + 3c_2 \\ 2c_2 & -c_1 + ac_2 \end{bmatrix}$

$$\Rightarrow \left.\begin{aligned} 2c_1 + 5c_2 &= 1 \\ c_1 + 3c_2 &= 1 \\ 2c_2 &= 2 \\ -c_1 + ac_2 &= 0 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} c_1 &= -2 \\ c_2 &= 1 \end{aligned}\right\} \Rightarrow \boxed{a = \frac{c_1}{c_2} = -2}$$