

Solve 1st order D.E. $\frac{dy}{dx} = f(x, y)$ analytically

Type	general form	Technique
Separable D.E.	$P(y) \frac{dy}{dx} = Q(x)$	$P(y) dy = Q(x) dx$ $\Rightarrow \int P(y) dy = \int Q(x) dx + C$
1st-order linear	$\frac{dy}{dx} + P(x)y = Q(x)$	$I(x) = e^{\int P(x) dx}$ $\frac{d}{dx}(Iy) = I Q(x) \Rightarrow y = \frac{1}{I} \left[\int I Q(x) dx + C \right]$
1st-order homogeneous	$\frac{dy}{dx} = f(x, y)$ where $f(x, y) = f\left(\frac{y}{x}, \frac{y}{x}\right)$	Change variables — $u = \frac{y}{x}$ to reduce the D.E. to a separable D.E.
Bernoulli Equation	$\frac{dy}{dx} + P(x)y = Q(x)y^n$	Change variables — $u = y^{1-n}$ after dividing by y^n to reduce the D.E. to a 1st-order D.E.
Exact D.E.	$M(x, y) dx + N(x, y) dy = 0$ with $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$	The solution is $\phi(x, y) = C$ where $\phi(x, y)$ is determined by $\frac{\partial \phi}{\partial x} = M$ and $\frac{\partial \phi}{\partial y} = N$