Instructions:

1. This package contains 14 problems, each worth 7 points, for a total of 100 points (that includes 2 bonus points for coming).

2. Please supply all information requested above and on the mark–sense sheet.

3. Work only in the space provided, or on the backside of the pages. Mark your answers clearly on the mark–sense sheet.

4. No books, notes, or calculator, please.
1. Let $f$ be a function defined on $(-\infty, \infty)$. Which is (are) true?

(I.) $f$ is continuous at $a$ if \( \lim_{x \to a} f(x) = f(a) \).

(II.) $f$ is continuous at $a$ if \( \lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) \).

(III.) If $f$ is differentiable, then $f$ is continuous.

A. Only I.
B. Only II.
C. Only I and III.
D. Only I and II.
E. Only II and III.

2. The numbers where the function sketched below is discontinuous are

A. 0, 1, 2
B. 1, 2
C. 1, 3
D. 2, 3
E. 0, 2
3. The horizontal asymptote of the function \( f(x) = \frac{8x + 6x^3}{12x^2 - 2x^3} \) is the line
   
   A. \( y = -3 \)
   
   B. \( y = \frac{2}{3} \)
   
   C. \( y = \frac{1}{2} \)
   
   D. \( y = 0 \)
   
   E. No horizontal asymptote exists.

4. \( \lim_{x \to \infty} \left( \sqrt{2x^2 + 4x} - \sqrt{2x^2 - 4x} \right) = \)
   
   A. 0
   
   B. \( \infty \)
   
   C. 4
   
   D. \( \sqrt{2} \)
   
   E. \( 2\sqrt{2} \)
5. If $f$ is given by this graph, which of the statements I, II and III is (are) true?

$$\begin{align*}
\text{I. } & f'(-2) < 0 \\
\text{II. } & f'(0) = 0 \\
\text{III. } & f'(2) > 0
\end{align*}$$

A. Only I and II. 
B. Only II and III. 
C. Only II. 
D. All are true. 
E. None are true.

6. If the tangent line to $y = f(x)$ at $(1, 2)$ passes through the point $(3, 2)$, then

A. $f(1) = 2, f'(3) = 2$
B. $f(2) = 1, f'(3) = 2$
C. $f(1) = 2, f'(1) = 0$
D. $f(1) = 3, f'(1) = 2$
E. $f$ is not differentiable at $(1, 2)$. 
7. Given the following graph of a function $f$, the graph of $f'$ would look most like

8. Let $y = e^{x^2 + 1}$. Then $\frac{dy}{dx}$ at $x = 2$ is
9. Let \( f(x) = x^3 \cos \left( \frac{\pi x}{2} \right) \). Then \( f'(1) = \)

A. \(-\frac{\pi}{2}\)  
B. \(3 - \frac{\pi}{2}\)  
C. \(\frac{3\pi}{2}\)  
D. \(-1\)  
E. \(\pi\)

10. Given \( f(x) = \frac{x - 1}{x + 1} \), then \( f'(x) = \)

A. \(\frac{2}{(x + 1)^2}\)  
B. \(\frac{2x}{(x + 1)^2}\)  
C. \(\frac{2}{x + 1}\)  
D. \(\frac{-2x}{x + 1}\)  
E. \(\frac{-2}{(x + 1)^2}\)
11. If \( f(x) = \frac{x^2}{g(x)} \), where \( g(2) = \sqrt{3} \) and \( f'(2) = 2 \), then \( g'(2) \) is

A. \( \frac{3 - \sqrt{3}}{2} \)
B. \( \sqrt{3} - \frac{3}{2} \)
C. \( \frac{3\sqrt{3}}{2} \)
D. \( \frac{-3\sqrt{3}}{2} \)
E. cannot be determined

12. Given \( f(t) = \frac{3}{\sqrt{t^2}} + 2\sqrt{t^3} \), \( f'(t) = \)

A. \( \frac{3}{2\sqrt{t^2}} + \sqrt{t} \)
B. \( \frac{2}{3\sqrt{t^2}} + \sqrt{t} \)
C. \( \frac{2}{3\sqrt{t}} + 2\sqrt{t} \)
D. \( \frac{2}{3\sqrt{t}} + 3\sqrt{t} \)
E. \( \frac{3}{2\sqrt{t}} + 3\sqrt{t} \)
13. A rotating beacon of light is located 1 km from a straight shoreline. See the figure below. What is the rate of change of $x$ with respect to $\theta$ (in km/rad) at $x = 2$ km?

\[
\text{Beacon} \\
\theta \\
1 \\
\text{beam of light} \\
\text{straight shoreline} \\
x
\]

A. 2  \\
B. $\sqrt{5}$  \\
C. $\frac{\sqrt{5}}{2}$  \\
D. 1  \\
E. 5

14. If $y = (\tan(x^4 + x))^3$, then $\frac{dy}{dx}$ at $x = 1$ is

A. $3(\tan^2 2)(\sec^2 2)$  \\
B. $5(\tan^2 2)(\sec^2 2)$  \\
C. $10(\tan^2 2)(\sec^2 2)$  \\
D. $15(\tan^2 2)(\sec^2 2)$  \\
E. $20(\tan^2 2)(\sec^2 2)$