1. A ladder 10 feet long is leaning against a wall. The foot of the ladder is being pulled away from the wall at 3 feet per second. How fast, in feet per second, is the top of the ladder sliding down the wall when the foot of the ladder is 8 feet from the wall?

   A. 1
   B. 2
   C. 3
   D. 4
   E. 5

2. A spherical tank has radius equal to 10 feet (= 120 inches). Use differentials to estimate, in cubic inches, the amount of paint needed to cover the surface with a layer \( \frac{1}{100} \) of an inch thick. \( V = \frac{4}{3} \pi r^3 \).

   A. \( 288\pi \)
   B. \( 480\pi \)
   C. \( 576\pi \)
   D. \( 640\pi \)
   E. \( 960\pi \)

3. Find the absolute minimum of the function

   \[ f(x) = 4x^3 - 15x^2 + 12x + 7 \]

   on the closed interval \([0, 3]\).

   A. 0
   B. 1
   C. 3
   D. 5
   E. 7
4. How many real roots does the equation \( x^7 + x + 1 = 0 \) have?

A. 1  
B. 2  
C. 3  
D. 5  
E. 7

5. Find the largest interval on which the function \( f(x) = x \sin x + \cos x \), \( 0 \leq x \leq \pi \), is increasing.

A. \((0, \pi)\)  
B. \((0, \frac{\pi}{2})\)  
C. \((\frac{\pi}{2}, \pi)\)  
D. \((0, \frac{\pi}{3})\)  
E. \((\frac{\pi}{3}, \frac{5\pi}{3})\)

6. What is the length of the largest interval on which the function \( f(x) = x^3 - 3x^2 - 9x \) is decreasing?

A. 1  
B. 2  
C. 3  
D. 4  
E. \(\infty\)
7. On what interval is the graph of the function
\[ f(x) = 1 - \frac{2}{x} + \frac{1}{x^2} \]
concave downward?

A. \((\frac{3}{2}, \infty)\)
B. \((1, \frac{3}{2})\)
C. \((\infty, 0)\)
D. \((0, 1)\)
E. \((1, \infty)\)

8. \(\lim_{x \to \infty} \frac{(\ln x)^3}{x^2} = \)

A. 0
B. 1
C. \(\frac{3}{2}\)
D. \(\frac{9}{4}\)
E. \(\infty\)

9. Given the following information about limits, select the graph that could be the graph of \(y = f(x)\).

\[ \lim_{x \to \infty} f(x) = \lim_{x \to -\infty} f(x) = 0, \quad \lim_{x \to -1^-} f(x) = \lim_{x \to 1^+} f(x) = \infty \]

\[ \lim_{x \to -1^+} f(x) = \lim_{x \to 1^-} f(x) = -\infty \]

A. 

B. 

C. 

D. 

E. 


10. The function \( f(x) = x^4 - 3x^3 + 3x^2 - x \) has critical numbers \( c = \frac{1}{4}, 1 \); indeed \( f'(x) = (4x - 1)(x - 1)^2 \). At these critical numbers \( f \) has

A. a local max. at \( \frac{1}{4} \), a local min. at 1
B. a local max. at 1, a local min. at \( \frac{1}{4} \)
C. a local max. at 1, neither a local max. nor a local min. at \( \frac{1}{4} \)
D. a local min. at \( \frac{1}{4} \), neither a local max. nor a local min. at 1
E. neither a local max. nor a local min. at either \( \frac{1}{4} \) or 1

11. Find the maximum value of the function \( \frac{x^2 + 2x - 4}{x^2} \).

A. \( \frac{1}{4} \)
B. \( \frac{9}{4} \)
C. \( \frac{7}{4} \)
D. \( \frac{3}{4} \)
E. \( \frac{5}{4} \)

12. A rectangular cardboard box of 32 in\(^3\) volume with a square base and an open top is to be constructed. Neglecting waste, find the minimum area of cardboard needed.

A. 54 in\(^2\)
B. 48 in\(^2\)
C. 46 in\(^2\)
D. 42 in\(^2\)
E. 40 in\(^2\)
13. Given the graph of \( y = f'(x) \) below, select a graph which could be the graph of \( y = f(x) \).

\[ \frac{y}{x} \]

A. \[ \frac{y}{x} \]  
B. \[ \frac{y}{x} \]  
C. \[ \frac{y}{x} \]  
D. \[ \frac{y}{x} \]  
E. \[ \frac{y}{x} \]

14. If \( f''(x) = 12x^2 + 2 \), \( f(0) = 2 \) and \( f'(0) = 3 \), find \( f(1) \).

A. 3  
B. 4  
C. 5  
D. 6  
E. 7